ARCHIVES

Problems with causal-loop diagrams

George P. Richardson

Founded in the mid-1950s, the field of system dynamics has intellectual roots reaching much further into the past. The Archives section of the Review seeks to publish material from that past which can contribute to current theory and practice. The section welcomes previously unpublished but deserving system dynamics work, classics from past system dynamics literature that should receive renewed attention, and previously published articles from other disciplines of particular significance to current system dynamicists. Contributions emphasizing the philosophy and theory of model building, validation, implementation, education, and generic structures are particularly encouraged. Submissions may range in length from notes to main articles but may be edited. Send suggested material, together with a brief introduction placing it in historical and current contexts, to John D. Sterman, System Dynamics Group, MIT E40-294, Cambridge, MA 02139, USA

The first system dynamics work did not include the use of causal-loop diagrams. Feedback structure was portrayed by equations or stock-and-flow diagrams. Such representations were natural for engineers. In an attempt to make system dynamics accessible to a wider range of people, causal-loop diagrams have become increasingly popular. In many texts and courses they are the first tool described. Indeed, recently several analysts have proposed that system dynamics studies can be carried out without the development of formal models at all (Morecroft 1985; Wolstenholme and Coyle 1983; Wolstenholme 1985). Causal-loop diagrams often figure prominently in such analyses. Yet even those who advocate the use of qualitative system dynamics are careful to point out that in all the successful applications of such qualitative methods the analysts have had extensive experience with formal model building. Nevertheless, it seems inevitable that people at all experience levels will continue to rely on causal-loop diagrams.

In the following paper dating from 1976, George Richardson describes a variety of problems which often arise in causal-loop diagramming, both in the development of the diagrams and the explication of behavior from them. The main difficulties arise because causal-loop diagrams obscure the stock and flow structure of systems. We sometimes emphasize so heavily the role of feedback structure in generating behavior that the crucial role of accumulation processes is lost. Even experienced modelers are easily misled by causal-loop diagrams. I suggest the following experiment: take the causal-loop diagram for the family feud described in Richardson's paper and ask a random sample of system dynamics modelers or students how it will behave. In my experience, one will not only receive a wide range of answers but most of these will be incorrect. Then repeat the experiment using the stock-and-flow diagram (with a different group of people, obviously). While answers will still vary, the number of correct responses should rise. In recognition of these difficulties, there has been a revival of stock-and-flow diagrams as a means of communicating structure (Morecroft 1982). Richardson's paper should not be taken as an argument to abandon causal-loop diagrams or qualitative system dynamics, however. But it serves as a caution to the facile use of an easily abused technique. Despite their problems, causal-loop diagrams are likely to remain important tools for the communication of feedback structure.

John D. Sterman, Editor

Introduction

Positive and negative feedback loops are the building blocks of system dynamics. While a complete specification of the feedback structure of a system requires specifying levels (states) and rates, the essential components and interactions in a system can be communicated quickly and concisely in a causal-loop diagram. The simplicity of causal-loop diagrams has led to their use in the early stages of model conceptualization, in introductory curriculum material in system dynamics from the fifth grade to graduate school, and in presentations of system dynamics studies in both technical and popular publications.

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The simplicity of causal-loop diagrams hides a subtlety, however, which poses problems which have not been adequately acknowledged.

The crux of the problem with causal-loop diagrams, which this paper explores in some detail, is that they make no distinction between information links and rate-to-level links (sometimes called "conserved flows"). That simplification is usually thought to be one of the advantages of causal-loop diagrams, but it has a rather dramatic disadvantage: in cases involving rate-to-level links the standard characterizations of positive and negative polarities in causal-loop diagrams are false.

This paper first exposes the difficulties in the traditional definitions of positive and negative links in causal-loop diagrams. Several possible improvements are suggested. Loops are then considered, and flaws in their traditional definitions and characterizations are uncovered, leading to the conclusion that definitions and characterizations in terms of dynamic behavior are not possible.

While the observations in this paper may have some significance for practicing system dynamicists, particularly in their writings for general audiences, the paper's major purpose is to clarify ideas important in the teaching of system dynamics. I should say at the outset that the basic ideas presented here are not new, most having appeared in some form previously. However, the seductive simplicity of causal-loop diagrams has led to general practice which is sometimes too casual and which may lead to misunderstandings.

Definitions of positive and negative links

Traditional definitions

The following definitions of positive and negative influences in causal-loop diagrams are representative of the general literature.[i]

> The following figure (Figure 1) shows a possible set of causal relationships. The arrows indicate the causal direction of influences. The signs adjacent to the arrows indicate the polarity. A plus (+) sign implies that a change in the variable at the end of the arrow will cause a change in the variable at the top of the arrow in the same direction. ... Similarly, a minus (-) sign implies that a change in the variable at the end of the arrow will cause a change in the variable at the top of the arrow in the opposite direction.

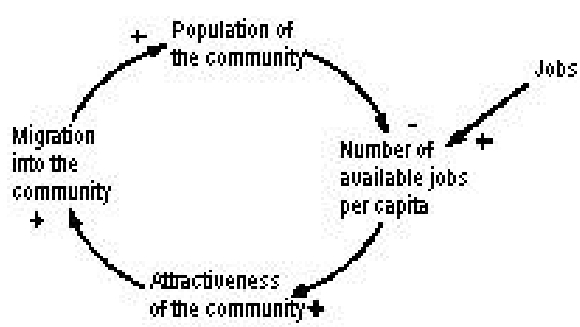


Fig. 1. A typical causal-loop (influence) diagram used to define positive and negative causal links (influences)

The arrow from "attractiveness" to "migration" is cited as an example of a positive influence: "An *increase* in the attractiveness of the community *increases* migration into the community." The arrow from "population" to "jobs available per capita" is given as an example of a negative influence: "An *increase* in the community's population will cause a *decrease* in the number of available jobs per capita." The definitions given in this reference in terms of changes of variables are entirely consistent with the examples given.

However, if the positive definition had been applied to another positive influence in the loop in Figure 1, an inconsistency would have appeared. Consider the link from migration to population. The definition claims that a change in migration will produce a change in population in the same direction, yet a *decrease* in migration will not produce a *decrease* in population unless migration becomes negative, drawing people out of the city. As long as migration is positive, it will always increase the population of the community, whether migration itself is increasing or decreasing. Furthermore, it is not even always true that an *increase* in migration produces an *increase* in population in the loop in Figure 1. Suppose available jobs per capita is so low that the community is not attractive and people are migrating out of the city. In such a case, the negative migration will always decrease the population of the community, whether migration itself is increasing or decreasing (provided, of course, that the increase in migration is not enough to make the net migration positive).

Thus, it can not be said with any certainty that a change in migration in the loop in Figure 1 will produce a change in population in the same direction. Part of the problem here is that migration can be interpreted to represent a *net* rate (see below), but the real difficulty is much more ubiquitous. The traditional definitions of positive and negative links in causal-loops fail for at least one link in most causal-loop diagrams system dynamicists might draw. In the simple positive loop involving population and births per year, the link from births to population fails the traditional definition: a decrease in births per year will not result in a decrease in population, since births can only increase a population. In the common illustrative negative loop representing the filling of a beer glass, the link from the rate of beer flow to the level of beer in the glass fails the traditional definition: here a decrease in the flow will not produce a decrease in the level of the beer in the glass. A host of other examples could be described, but the point is clear: the traditional definitions of positive and negative links fail in a wide variety of cases.

The source of the problem

The reason that each of these links is inconsistent with the traditional definitions is that each represents a rate-to-level connection (a "conserved flow"), while the definitions are applicable only to information links. For a conserved flow, the variable at the tail of the arrow is the rate of change (the derivative) of the variable at the head of the arrow. (In some of the loops mentioned, the variable at the tail is only the positive or negative part of the derivative, e.g., births per year, deaths per year, and so on, but the importance of the observation remains the same.) It is an elementary notion of calculus that the increasing or decreasing nature of the derivative $f'(t)$ determines the curvature of the graph of $f(t)$, not whether $f(t)$ is itself increasing or decreasing.

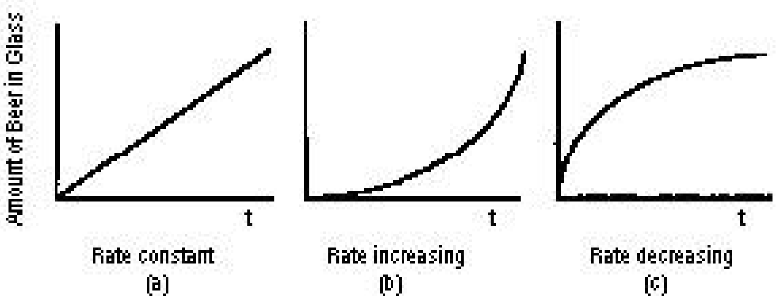


Fig. 2. Patterns of behavior of an accumulation over time (the level of beer in a glass) affected by different inflow rates, illustrating that the increase or decrease in the inflow rate affects only the *curvature* of the graph of the accumulation, not whether the accumulation itself increases or decreases

For example, consider filling a glass of beer. If the rate of flow into the glass is constant (and positive), the level of beer in the glass over time has the linear graph shown in Figure 2a. If the rate of flow is increasing, the level of beer in the glass over time has the upwardly curving graph shown in Figure 2b, and the beer glass is filling faster and faster (a dangerous policy). If the rate of flow is decreasing (but still positive), the level of beer over time would appear as in Figure 2c, where the fill rate sensibly slows as the level in the glass gets higher.

The graphs in Figure 2 show that the increasing nature of the level of beer is not changed by an increase or decrease of the fill rate; only the curvature changes. The traditional definition fails in the case of this positive link from the fill rate to the level of beer in the glass precisely because the link represents a conserved flow. It should be clear that three similar, but decreasing graphs could be drawn for a negative rate-to-level link such as the one from deaths per year to population, and a similar conclusion results. The traditional definitions work for links that represent proportional relationships, but fail in every case representing accumulations of a rate of flow.[ii]

Improved definitions

A possible improvement in the traditional definition of a positive influence in a causal-loop diagram is the following, which is suggested by the preceding observations about curvature:

> *A* has a positive influence on *B* if an increase (decrease) in *A* results in a value of *B* which is greater (less) than it would have been had *A* not changed.

(A similar definition for a negative influence is easy to invent.) In the beer glass example, an increase in the rate of flow results in a higher level of beer in the glass than would have resulted had the rate stayed constant (compare Figures 2b and 2a). Similarly, a decrease in the rate of flow results in a lower beer level than would have occurred had the rate stayed constant (compare Figure 2c and 2a). A check shows that the definition behaves properly in all instances, for information links as well as rate-to-level connections.

Although the definition problem is apparently solved by phrasings like the above, there are undoubtedly instances in which one would not want to use such a definition.

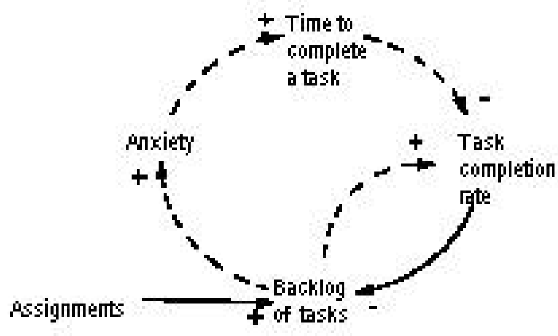


Fig. 3. A causal-loop diagram distinguishing between additive (rate-to-level) links and proportional (information) links. Solid arrows are used here to identify real addition and subtraction processes

In introductory course material such definitions might initially obscure more than they clarify. They might also prove too cumbersome in descriptive writings about system dynamics studies. Furthermore, the suggested definition tends to hide the reason why the additional wording is necessary. A second attempt is the following, which acknowledges the distinction between information links and conserved flows which is at the heart of the problem:

> *A* has a positive influence on *B* if *A* adds to *B*, or if a change in *A* results in a change in *B* in the same direction.

Similarly,

> *A* has a negative effect on *B* if *A* subtracts from *B*, or if a change in *A* results in a change in *B* in the opposite direction.

The distinctions made in these definitions is the same one made by different arrows used in rate/level flow diagrams, in which solid arrows are commonly used for rate-to-level connections (adding or subtracting) and dotted arrows are used for information links (Forrester 1961, 67-72). These definitions suggest that causal-loop diagrams might be improved if two different symbols were used, acknowledging the two kinds of links. For example, in Figure 3, solid and dotted arrows represent, respectively, conserved flows and information links.

Some authors naturally make analogous distinctions in the causal-loop diagrams they display for general readership. In one subtle but effective variation, straight lines are used for rate-to-level connections, while curved arrows are reserved for information links. The diagrams are no more visually complicated than traditional causal-loop diagrams, but they significantly help the sensitive reader to discern the real structure of the assumptions in the diagram. A more common technique is exemplified by the diagrams in Levin, Roberts, and Hirsch (1975) in which rates and levels are explicitly represented as "valves" and "tubs" while all other connections are the curved solid arrows of traditional causal-loop diagrams. Each of these variations is a conscious or unconscious attempt to deal with the difficulties resulting from representing rate-to-level links in causal-loop diagrams.

There are other ways of pursuing the perfect definition for the polarities of causal links, acknowledging in some way the distinction between a flow and an information link. The reader may have his own favorite among these improved definitions. Perhaps different definitions suit different audiences and purposes. There may even be situations in which the common, flawed definitions are the most advisable. It is the author's belief, however, that a sufficient number of confusions can arise from the common definitions in introductory courses using causal-loop diagrams that an improved definition such as the second one given above is advisable. A definition drawing the distinctions between additive and proportional links (between conserved flows and information links) helps to move a student from elementary conceptualizations to modeling concepts. Furthermore, it increases the likelihood that a simple causal-loop diagram can be "read" correctly and its dynamic behavior to some extent inferred. Unfortunately, while improving the "readability" of feedback loops, the recognition of rate-to-level links in causal loops invalidates some of the traditional definitions of the polarity of causal loops, as the following sections show.

Characterizing positive and negative causal loops

Because system dynamics involves the study of the relationships between feedback structure and dynamic behavior, there is a great impetus to try to infer dynamic behavior from representations of structure. That impetus has apparently led to a set of definitions of the polarities of causal-loops which are phrased in terms of behavior over time. As with definitions of causal links, so it is with causal loops: the existence of rate-to-level links invalidates the traditional definitions of positive and negative loops in causal-loop diagrams. In this section the difficulties with the traditional definitions are noted, with the discussion focusing particularly on rate-to-level links, hidden loops, and net rates. The section ends with a brief analysis of a causal-loop which, together with the rest of the paper, casts grave doubt on the possibility of defining the polarity of causal loops in terms of dynamic behavior.

Common definitions

A positive loop is often defined "...by the fact that an initial change in any factor eventually induces further self-change in the original direction"(Levin, Roberts, and Hirsch 1975, 7). Representative of the definitions of negative feedback loops is the following: "When a feedback loop response to a variable change opposes the original perturbation, the loop is negative or goal-seeking" (Goodman 1974, 9). The definition of a negative loop is usually interpreted to mean that "...a change in one element is propagated around the circle until it comes back to change that element in a direction opposite to the initial change" (Meadows 1972, 42).

These definitions lead nicely to the reliable characterizations of a positive feedback loop as a loop having an *even* number of negative causal links, and a negative loop as one having an *odd* number of negative links. In causal-loop diagrams drawn without rate-to-level links, these definitions are completely consistent with the traditional definitions of positive and negative links discussed in the first section of this paper. Figure 4 shows examples of a positive loop and a negative loop which are consistent with these definitions, as the reader can verify by tracing a change in some variable around each loop.

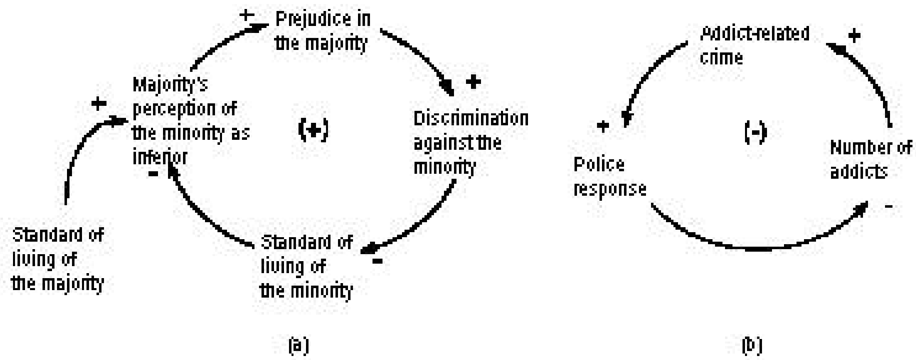


Fig. 4. Typical causal-loop diagrams used to illustrate definitions of loop polarity

It is interesting to note for later reference that these definitions have two particular implications. First, a positive loop consistent with the definitions can be traced as either an increasing loop or a decreasing one—in Figure 4a one would talk about the loop as a "vicious cycle" or a "benign" one depending on the direction of the initial exogenous change. Second, while no one intends this conclusion, every negative loop fitting the causal-loop definition is, by implication, an oscillating structure—an increase, so the definition says, traced around the loop becomes a decrease, which produces an increase after another cycle, and that a decrease, and so on, up and down through time. Oscillations, however, depend on rate/level structure as well as feedback structure. The fact that not every positive loop and negative loop behave in these ways suggests that there are problems with the traditional definitions of the polarity of causal loops, as the following sections on rate-to-level links, hidden loops, and net rates show. The most persistent problem here is the urge to define polarities in terms of dynamic behavior: because behavior depends upon rates and levels, unspecified in causal-loop diagrams, universally applicable definitions in terms of behavior appear to be most difficult to invent.

Rate-to-level links

In any loop involving an explicit rate-to-level link, the traditional definitions of the polarities of feedback loops given above produce inconsistencies. As an example, consider the elementary population/births loop shown in Figure 5. Tracing the implications of an increase in population, no real difficulties arise; the initial change induces “further self-change in the same direction,” and the loop fits the common definitions of a positive loop. Suppose, on the other hand, that the initial change is negative, there is a decrease for some reason in population. Births per year will decrease, but births will still increase population (though less rapidly) because the link from births to population is a "flow," a rate-to-level link. Thus, an initial decrease in population can not be said to induce further self-change in the same direction. (Note that if the rate-to-level link is read incorrectly—a decrease in births leads to a decrease [*sic*] in population"—the loop fits the definition again. There is a natural human tendency, it seems, to be more consistent than accurate when one can't be both.)

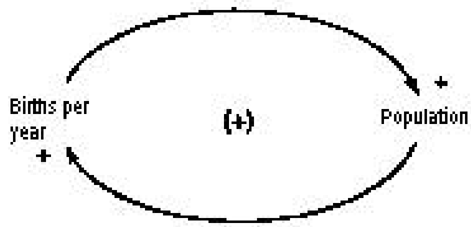


Fig. 5. A positive feedback loop that fails the usual definition of positive loop polarity because of an explicit rate-to-level link

Similarly, difficulties appear in negative loops with explicit rate-to-level links. In the common population/deaths loop, for example, a decrease in population is not "...propagated around the circle until it comes back to change that element in a direction opposite to the initial change." Clearly, not every positive feedback loop can be meaningfully traced as both an increasing and a decreasing loop (both "vicious" and "benign"), and not every negative loop is an oscillator, as the traditional causal-loop definitions imply. The usual definitions in terms of behavior over time assume that all links in the loop are of the proportional kind rather than the accumulating, rate-to-level kind.

Hidden loops

Underscoring the difficulty of defining polarities of causal-loops in terms of behavior are feedback systems symbolized with hidden (unrepresented) feedback links. An instructive example is the following salesman loop from *Principles of Systems* (Forrester 1967, 2-21 to 2-25). It is common practice, even in stock-and-flow diagrams, to omit the rate and the minor negative loop in such exponential smoothing structures, showing only the basic positive loop summarized in Figure 6b. The hidden loop, however, has much to do with how such a system behaves over time, thus complicating the problem of trying to define the polarity of a causal loop in terms of behavior. Whether the salesforce increases, stays constant, or decreases depends upon whether "indicated salesforce" is greater than, equal to, or less than "salesforce," since

$$\text{hire/fire rate} = \frac{\text{indicated salesforce} - \text{salesforce}}{\text{time to adjust salesforce}}$$

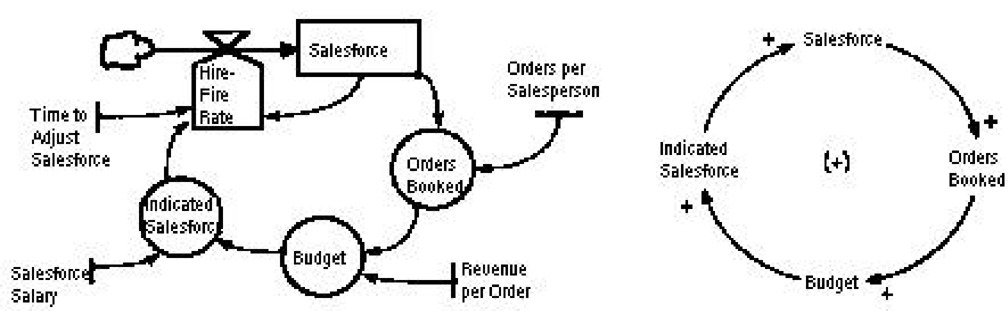


Fig. 6. Different representations of feedback loops affecting the growth of a salesforce, illustrating hidden loops and net rates in causal loop diagrams

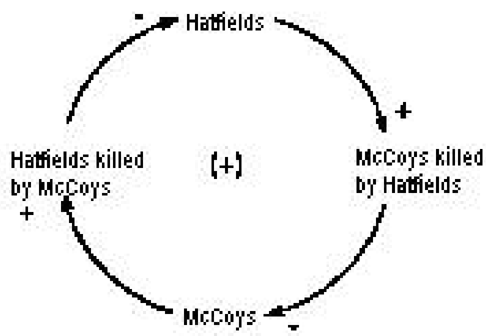
The relationship between "indicated salesforce" and "salesforce" is solely determined by the values of three parameters appearing in Figure 6a but always omitted and unspecified, as in Figure 6b, in a causal-loop diagram of such a system.[iii] The salesforce loop is positive because "indicated salesforce" enter the rate equation with a positive sign, not because the loop responds to a change in a certain way. A definition of a positive-loop which is phrased in terms of the increase of variables in the loop is difficult to apply—it can not be said with any certainty that a change in any one of the variables in the causal loop can be traced around the loop to produce "further self-change in the same direction." Of course, one could (and should) invoke a *ceteris paribus* stance when defining the polarity of a causal-loop, but because of loops like the salesforce loop one is still faced with definitions of behavior that might not match the actual behavior of the system because of hidden loops.

Net rates

The salesforce loop in Figure 6 contains a net hire/fire rate, a concept causing further difficulties for causal-loop diagrams. Because a net rate is the aggregation of an inflow and an outflow, it is not possible to decide whether a net rate-to-level link is a positive influence or a negative one. Consider the net birth rate in an aggregated population model. If net births per year is positive, births add to population and the loop is a positive loop; conversely, if net births per year is negative, net births subtract from population and the loop is a negative loop identical to a population/deaths loop. Thus, a net rate in a causal-loop creates an undecidable loop—perhaps positive and perhaps negative, depending upon parameters and variables not contained in the loop.

It is interesting to note that the link from a net rate such as net births per year is commonly thought of and symbolized as a positive link, perhaps because the word "births" biases the case, but more likely because the population/births loop behaves like a positive loop when the link is erroneously thought of as an information link rather than a rate-to-level link. "If net births per year increase," we say, "then population increases, and if net births decrease then population decreases [*sic*]." While such a statement fits the pattern of the traditional definitions, we know that net births will decrease population only if the number declines far enough to be negative, in which case the loop is not positive, but negative. Stock-and-flow diagrams handle such a case with a double-headed arrow, but no similar convention has been established for causal-loop diagrams, presumably because the traditional definitions have obscured the need.

Fig. 7. Causal-loop diagram of a feud between the Hatfields and the McCoys



A final example

The presence of rate-to-level links, hidden loops, and net rates in causal-loop diagrams makes it difficult to define polarity in terms of behavior. Treating all links as proportional links guarantees that most inferences of behavior from causal-loop diagrams will be incorrect. However, recognizing rate-to-level links as well as information links makes it much more likely (though not certain) that dynamic behavior can be correctly inferred from causal-loops. The following example shows the greater reliability obtained by reading rate-to-level links correctly, while reiterating the inadequacies of the traditional definitions and emphasizing the dangers of attempting to infer dynamic behavior from the polarity of causal-loops.

Consider a family feud. The Hatfields and the McCoys have finally decided to have it out: Hatfields shoot McCoys, McCoys shoot Hatfields, and neither side has time to send for cousins from the next ridge to help. Applying the traditional definitions to the loop shown in Figure 7, we would conclude that an increase in Hatfields, for example, would lead to more McCoys being killed by Hatfields, which would lead to fewer McCoys and fewer Hatfields being killed, which would lead to more Hatfields [*sic*]—an increase in Hatfields leading to a further increase in Hatfields. A decrease in Hatfields could be similarly traced and the conclusion would be the same—one family increases and one decreases, presumably until it gives up or is wiped out. But it is silly, of course, to conclude that either family increases as a result of shooting each other.

Taking account of the conserved flows in the loop in Figure 7, we conclude that an increase in Hatfields leads to more McCoys being killed, which leads to a decrease in the number of McCoys and consequently a decrease in the number of Hatfields killed by McCoys (as before), which leads nonetheless to a *decrease* in the number of Hatfields. An increase in Hatfields leads to a decrease in Hatfields—such a statement is partly misleading. Continuing the cycle, we conclude that the decrease in Hatfields results in fewer McCoys killed by Hatfields, which results in still fewer McCoys but the drop is not as great now as it was in the previous cycle. Fewer McCoys kill fewer Hatfields, so the Hatfields decrease, but not as much as they did the first time around the loop. The loop says that both Hatfields and McCoys are decreasing, but decreasing less and less rapidly as time goes on. We might expect their graphs to look like those in Figure 8. The system appears to be goal-seeking, with behavior characteristic more of a negative loop than a positive one.

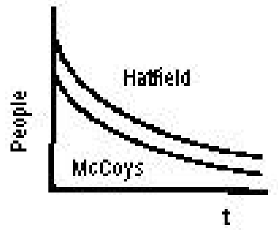


Fig. 8. Apparent goal-seeking behavior that can be inferred from the conserved flows implicit in the feud loop (Fig. 7)

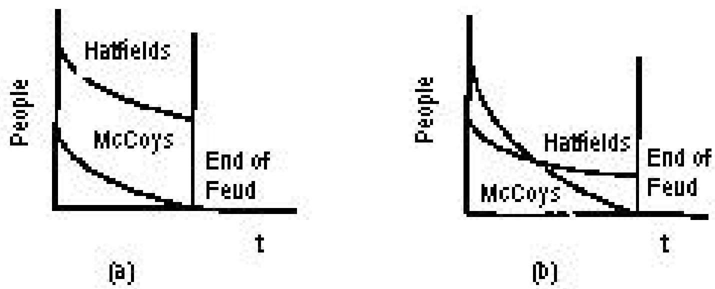


Fig. 9. More typical feud scenarios in which one side or the other is wiped out, illustrating the disequilibrating character of the positive feud loop

A bit more analysis shows that the Hatfields/McCoys loop has more positive characteristics than negative ones. Suppose the Hatfields hopelessly outnumbered the McCoys. The feud would be over rather quickly, and the graphs might look something like Figure 9a. Or suppose each Hatfield had a machine gun while the most the McCoys could muster was ten slingshots for the whole group. Figure 9b shows what we would expect.

Thus, the graphs in Figure 8, which exhibits goal-seeking behavior characteristic of a negative loop, are seen to suit a rather special case in which numbers and firepower balance out. Without additional negative feedback loops in the model to prevent Hatfields or McCoys from becoming negative, the scenarios in Figure 9 would continue over time to show McCoys becoming more and more negative and Hatfields growing! The loop indeed has the destabilizing character we associate with positive feedback loops, but not over the meaningful time period of the feud and not for some initial conditions.[iv]

The Hatfields and McCoys example shows quite clearly the extreme difficulty of defining the polarity of causal-loops in terms of the behavior a loop is supposed to exhibit in response to changes in its variables. The feud example also shows that predicting behavior from loop polarity alone without regard for distinction between rate-to-level links and information links is impossible.

Conclusion

The traditional definitions of the polarities of causal links and loops are inadequate. With slight modifications taking account of the accumulating nature of rate-to-level links, the traditional definitions of causal *links* can be corrected, with the recognition of conserved flows enhancing the "readability" of a causal loop. The urge to define the polarities of causal *loops* in terms of behavior over time, however, must be resisted. In particular examples, the destabilizing nature of positive feedback loops and the goal-seeking character of negative loops can be seen by tracing around the loop a change in one of its variables. In general, however, the results of such a change can not be stated with any certainty and with any universal applicability to all positive or negative loops. The best course of action appears to be to define clearly the polarity of causal-loops in terms of the number of negative links in a loop and to let intuitions about the dynamic implications of those polarities grow as more and more examples are seen and understood.

The difficulties of teasing dynamic behavior out of causal-loop diagrams suggest that people wishing to construct meaningful dynamic models should either avoid them or use them exceedingly carefully. Since to "read" them with any reliability requires a recognition of rate-to-level links, a modeler conceptualizing a system might just as well use a representation which better acknowledges stock-and-flow structure. While causal-loop diagrams may have a defensible role in elementary teaching in system dynamics, their most appropriate place appears to be in expository writing for public consumption. In such contexts, descriptions of dynamic behavior couched in causal-loop terms are backed up by the modeler's certain knowledge of how an actual dynamic model behaved when simulated or solved analytically.

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Notes

[i] The example cited is from Henize (1971). See also Goodman (1974, 7& 9), Meadows (1972, 41-42), Maruyama (1968, 81-82), and Roberts (1975, 2).

[ii] Milsum (1968, 29) distinguishes between an accumulating influence and one that is proportional.

[iii] Underlying these observations is the notion of the open-loop, steady-state gain of the positive loop. The salesforce level increases, stays constant, or decreases if the gain of the positive loop is, respectively, greater than, equal to, or less than 1. Statements about such gain are really statements about loop dominance. If the gain is greater than 1, the positive loop dominates, and if the gain is less than 1, the hidden negative loop dominates.

[iv] If the system shown in Figure 7 is given by

$H'(t)= -aM$ and $M'(t)= -bH$, $(a, b > 0)$

where H and M represent numbers of Hatfields and McCoys, then H(t) and M(t) show pure exponential decay if and only if $H_0(a)^{1/2} = M_0(b)^{1/2}$, then H(t) and M(t) are eventually dominated by terms involving exp $((ab)^{1/2})$, so the system explodes.