CHAPTER 13

LEVELS AND RATES

DIAGRAMING LEVELS AND RATES

The first step in moving from a causal-loop representation to a computer simulation model is the identification of system levels and rates. Recall from Part III that a level is a quantity that accumulates over time, and a rate is an activity, or movement, or flow that contributes to the change per unit of time in a level. For example, the number of cars in the Midtown Parking Lot is a level, and the number of cars arriving per hour is a rate. Similarly, the number of children at Hometown Elementary School sick with the flu is a level, while the number of children recovering per day is a rate. Population is a level, and the number of babies born per year is a rate.

In identifying a system's levels and rates, it is generally helpful to represent the system in flow diagram form. Figure 13.1 depicts the symbols that are used to represent levels and rates in flow diagrams. A level is depicted by a rectangle (which is supposed to resemble a box or a bathtub), and a rate is depicted by a symbol that looks somewhat like a valve. (A rate might be thought of as a faucet, controlling the flow of water into the bathtub.) A complete set of flow diagram symbols is included at the end of Chapter 15.

Figure 13.2 shows that the number of cars in the Midtown Parking Lot is influenced by the number of cars arriving per hour (the arrival rate). The flow of cars arriving at the parking lot increases the level of cars in the lot, much as the flow of water into a bathtub increases the level of water in the tub.

Figure 13.3 is a somewhat more complicated diagram, indicating that the number of children at the Hometown Elementary School sick with the flu is influenced by both the number of children catching the flu each day and the number recovering. The flow of children catching the flu—the infection rate—adds to the number of children sick with the flu; and the flow of children recovering—the recovery rate—subtracts from the number who are sick. (The situation is somewhat similar to a bathtub with both a faucet and a drain. The flow of water into a bathtub adds to the amount of water in the tub, and the flow of water out of the drain subtracts from the water in the tub.)

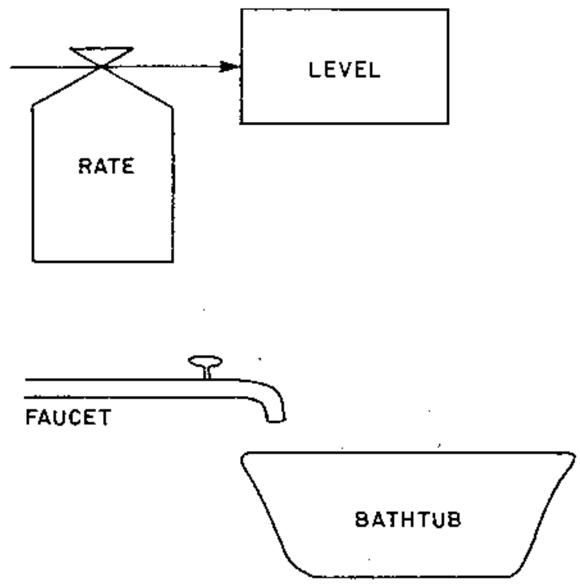


Figure 13.1 Flow diagram with level and rate

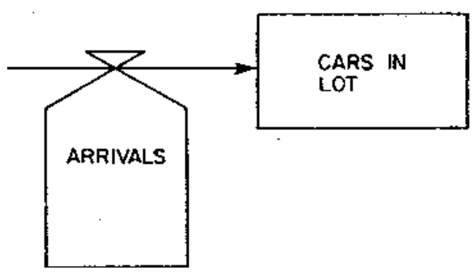


Figure 13.2 Midtown Parking Lot

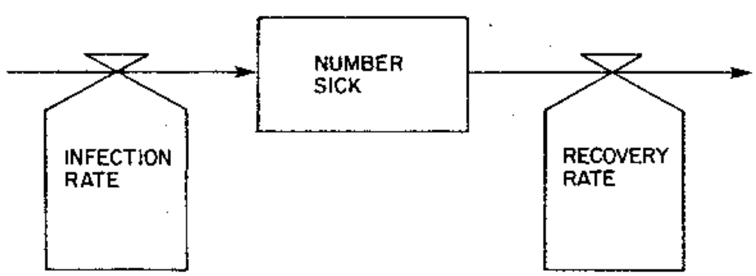


Figure 13.3 Hometown Elementary School

Exercise 1: Flow Diagrams

a. Draw flow diagrams for the following situations:
   1. The population of rabbits is influenced by the number of rabbit births per year.
   2. The number of yeast cells in a sugar solution is influenced by the number of buds formed per minute.
   3. A child's knowledge is influenced by his or her learning rate.

b. What rates influence the population of Boston? (Draw a flow diagram including whatever rates you think might be appropriate.)

c. What rates might influence the number of students enrolled in an urban high school? (Draw a flow diagram.)

d. Add the number of students susceptible to the flu and the number of children who have recovered from the flu to the Hometown Elementary School flow diagram, shown in Figure 13.3.

FROM CAUSAL LOOPS TO FLOW DIAGRAMS

Moving from a causal loop diagram to a flow diagram requires a few additional symbols. Figure 13.4 depicts a causal-loop diagram and a corresponding

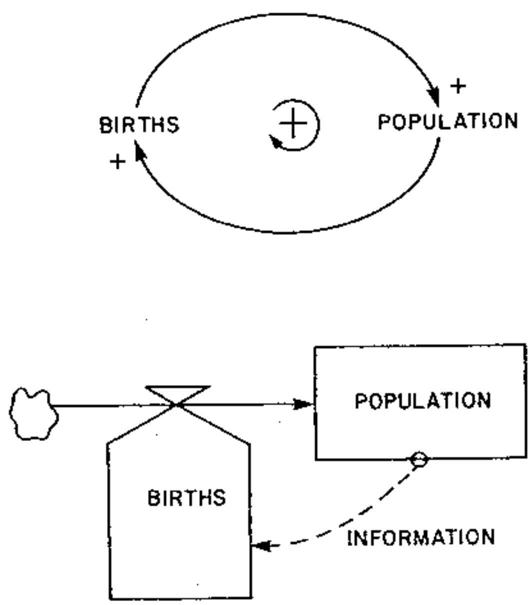


Figure 13.4 Causal-loop and flow diagrams of population and births

flow diagram of the interaction of population and births. The level in this instance is population, as indicated by the rectangle, and births is a rate, as indicated by the valve symbol. The positive link from births to population in the causal-loop diagram is depicted in the flow diagram as the flow of births into population. The direction of the solid arrow in the flow diagram indicates that births add to the population. The positive link from population to births, in the causal-loop diagram, is shown as a dotted line in the flow diagram, indicating that the size of the population influences the birth rate.

The "cloud" at the tail of the solid arrow represents the "source" of people. (Sources represent systems of levels and rates outside the boundary of the model. In this case, the source allows bypassing the issue of where babies come from!) Although not shown on this diagram, "clouds" can also be used to show "sinks," where flows terminate outside the system.

The flow diagram is a more detailed representation of the positive feedback loop than is the causal-loop diagram. It identifies population as a quantity that accumulates, and it identifies births as a quantity that influences how rapidly the population accumulates. The solid arrow shows the flow of people into population. The dotted arrow shows that the size of the population affects births, or that there is a cause-and-effect link from population to births. The causal-loop diagram ignores the distinction between a rate of flow and a cause-and-effect link not involving a rate of flow, but the flow diagram calls explicit attention to this distinction.[1]

Exercise 2: Population and Deaths

Figure 13.5 shows a causal-loop diagram of the interaction of population and deaths.

Identify the level and rate in the system, and draw a flow diagram.

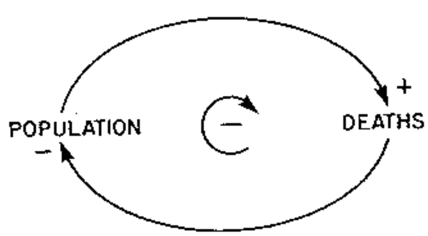


Figure 13.5 Causal interaction of population and deaths

Exercise 3: Natural Resources

Figure 13.6 depicts a causal-loop diagram of an interaction of natural resources and usage. The diagram depicts how a decreasing supply of a particu-

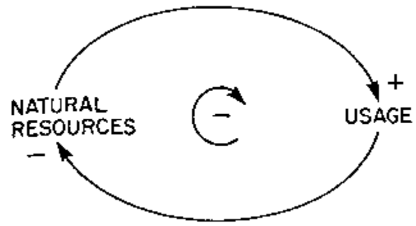


Figure 13.6 Causal-loop diagram of natural resources and usage

lar natural resource can result in less use of the resource, because it is harder to find.

Draw a flow diagram of this interaction. Begin by identifying which quantity is a level and which is a rate. Assume that no new quantities of the resource are created, so that there will be no source and no inflow. Treat the place where the used resource goes as a sink.

EXAMPLE I: CHILDREN AND ADULTS

Figure 13.7 depicts another causal-loop diagram of the growth of population through births. However, in this diagram population is separated into adults (individuals mature enough to bear children) and children (individuals too young to bear children).

To draw a flow diagram based on this causal-loop diagram, it is easiest to begin by identifying the levels and rates. Children and adults are levels, since they are quantities that accumulate over time; while births and children maturing are rates. (Note that the rates have units "People per Year," whereas the levels have units "People.") A partial flow diagram is shown in Figure 13.8. Births flow into the population of children, as indicated by the positive link from births to children in the causal-loop diagram. Furthermore, the flow of children maturing decreases the level of children and increases the level of adults. (Thus the flow of children maturing incorporates two links from the

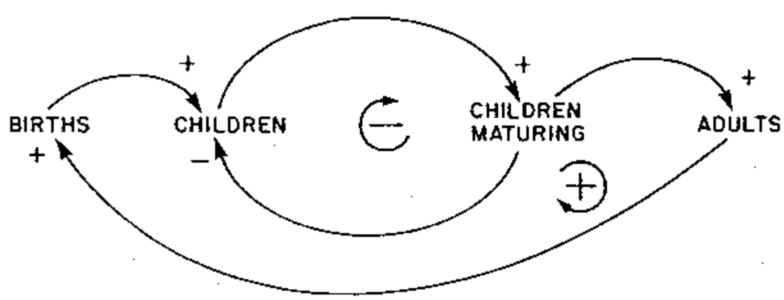


Figure 13.7 Causal-loop diagram of children, adults, and births

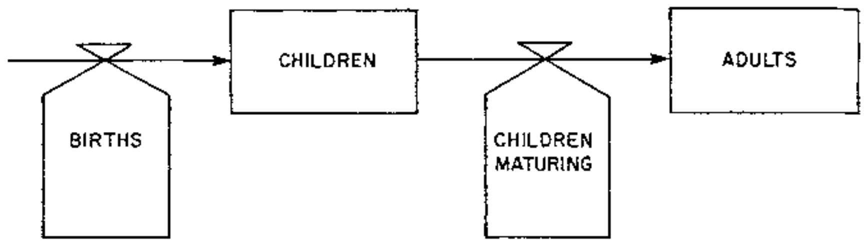


Figure 13.8 Levels and rates for children and adults

causal loop diagram: the negative link from children maturing to children, and the positive link from children maturing to adults.)

The next task in completing the diagram is to add the cause-and-effect links. For example, a positive link connects adults to births, since the more adults there are, the more births there will be (other factors remaining equal). This link is shown in the flow diagram as a dotted line running from the level of adults to the rate of births. One link in the causal-loop diagram remains to be inserted in the flow diagram—the positive link connecting children to children maturing. As this link indicates, the more children there are, the more children will mature. This link is represented in the flow diagram by a dotted line connecting the level of children to the rate of children maturing. Finally, the flow diagram is completed by adding a source symbol to the left of the births, as shown in Figure 13.9.

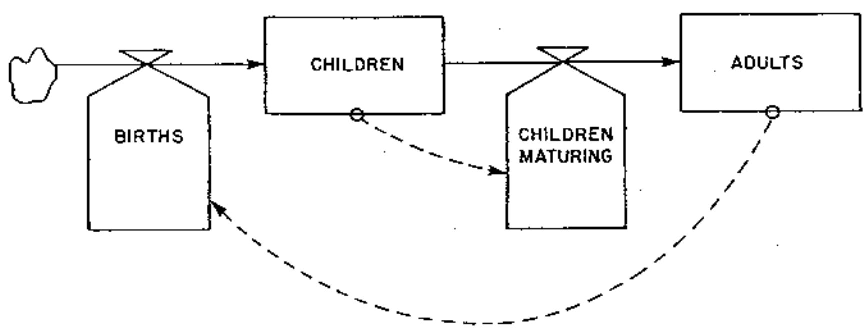


Figure 13.9 Completed flow diagram for children and adults

Exercise 4: Resource Processing

Figure 13.10 depicts a causal-loop diagram of the life cycle of aluminum used in cans. As aluminum is refined, it passes from the stage of being ore to being aluminum in process. The metal is then made into cans. The cans have an average life, after which they become solid waste. At each stage, the flow into the next stage depends on how many cans are at the current stage.

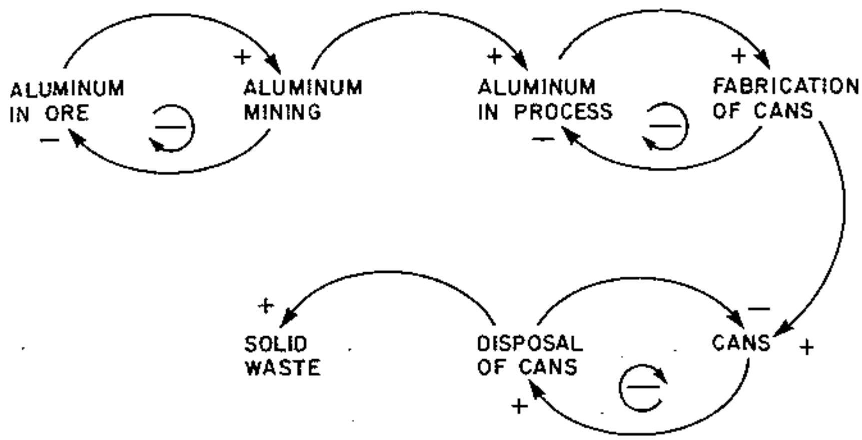


Figure 13.10 Causal-loop diagram of lifecycle of aluminum

Draw a flow diagram based on the causal-loop diagram. What would you add to your flow diagram to represent a recycling program?

SIMULATION, STRUCTURE, AND BEHAVIOR

The main reason for moving from a causal-loop representation of system structure to a flow diagram is to provide additional insight into the behavior a proposed model generates over time. For example, does the hypothesized model generate continued growth? If so, how rapid is the growth? Or, does the model generate decline? If so, how precipitous? Does the model exhibit goal-seeking behavior? If so, do model variables approach equilibrium smoothly, or do they oscillate? If the model produces oscillations, what is the period from peak to peak? How dramatic are the cycles? And so on.

In order to provide full answers to these questions, it is necessary to move one final step and express each model relationship in equation form. Of course, the translation from a verbal description of each model relationship to a statement as an equation often requires a good deal of ingenuity. However, in drawing out the implications of a model, equations are essential.

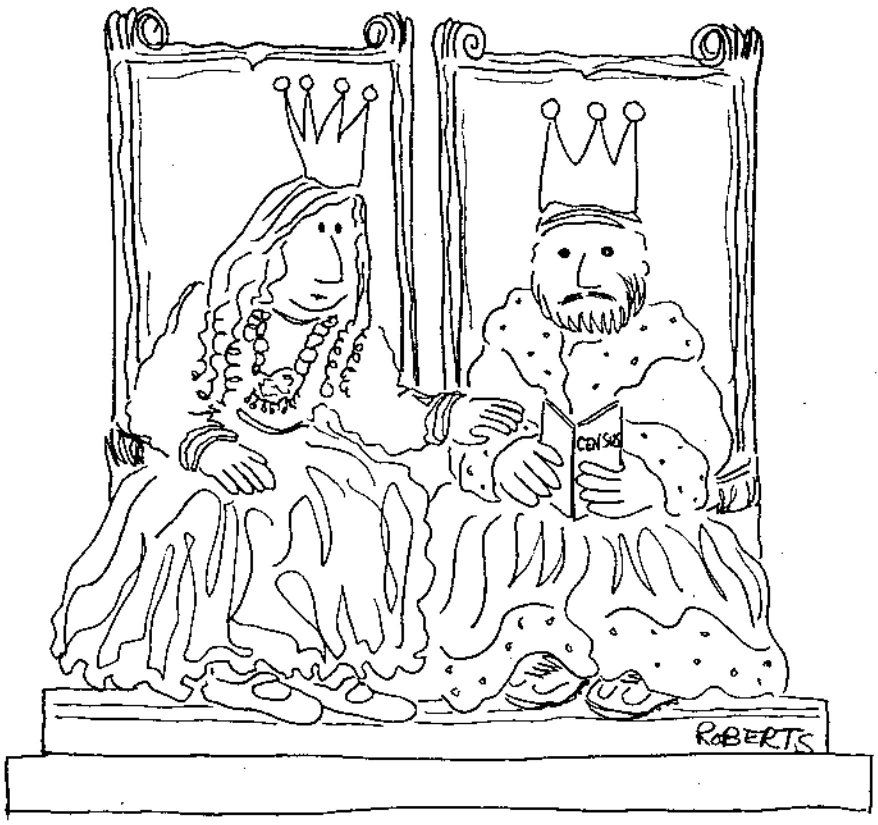
The strategy generally followed in formulating a model is to begin with a causal-loop diagram, then formulate a flow diagram, then write equations, and finally, use the equations to simulate the model on the computer. Once a "running" model has been developed, it can then be used to explore the consequences of alternative model assumptions and proposed policy interventions. Indeed, one of the main advantages of simulation is the opportunity it provides to move quickly and easily from one set of assumptions to another.

In the discussion of equation writing and simulation in the next few chapters, a good deal of attention is given to ways of using simulation to draw out the implications of hypothesized system relationships. Much less attention is given to methods of using empirical evidence to choose numerical values for model parameters. Nor is much attention given to methods of assessing the "match" between the behavior generated by a simulation model and the historical behavior of the actual system under study. This is not because these questions are unimportant or easy—they are not. Often, however, a good deal can be learned about a system by exploring the implications of alternative hypothetical models. In addition, in estimating model parameters and assessing the match between model behavior and historical evidence, it is worth paying a fair amount of attention to the relationship between the structure of a proposed model and the behavior it generates.

EQUATIONS FOR LEVELS AND RATES

Once a flow diagram has been developed, the next step in building a model is to write equations. The following examples introduce the general ideas involved. Chapter Fourteen then provides more detailed information on equation-writing using the DYNAMO computer simulation language.

EXAMPLE II: THE KINGDOM OF XANADU



In the mythical kingdom of Xanadu, exactly 100 babies are born every year, and no one ever dies. In last year's census (the year 2020, according to the Xanaduian calendar), the population was found to be 5510 people. Everyone in Xanadu believes that births will continue in the future as they have in the past.

The king of Xanadu wishes to have a model that will estimate the population of the kingdom for the next twenty years (the years 2020 through 2040). What will such a model contain? First, the model will contain variables, things whose numerical values change over time. As a notational convention, we will always refer to model variables using names written in ALL CAPITAL LETTERS. In the model for the king of Xanadu, population and births are variables, and for convenience they can be called POP and BIRTHS. POP, of course, is a level, and BIRTHS is a rate, as indicated in Figure 13.11.

The second thing a model must contain is a set of rules for computing the values of variables. For example, from the preceding description it is clear that the rule for births in Xanadu would be:

Set births equal to 100 people per year

An equation is a concise way of specifying a rule for computing a variable. For example, the rule for births in Xanadu could also be written in equation form:

BIRTHS = 100 people per year

This is called a "rate equation," naturally enough, since it is the equation for BIRTHS, which is a rate.

Now, how can an equation be written for the level of population over the twenty-year period from 2020 to 2040? The simplest approach is to break up the twenty-year period into one-year intervals, and then calculate the population year by year. In the year 2020, according to the Xanaduian calendar, the population was 5510. So, the first year that needs to be calculated is the year 2021. Recall that, in Xanadu, no one ever dies, nor does anyone enter or leave the Kingdom. Hence, the only change in the population from one year to the next is the number of new babies born—which is exactly 100. Thus the population in 2021 is just the population in 2020 plus 100.

POP(2021) = POP(2020) + 100 = 5510 + 100 = 5610

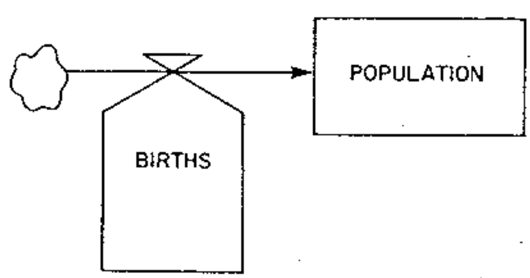


Figure 13.11 Flow diagram for Xanadu

Using this same procedure, it is easy to calculate what the population will be in the year 2022. The population in the year 2022 is simply the population in 2021 plus 100.

POP(2022) = POP(2021) + 100 = 5610 + 100 = 5710

The same idea can be used to simulate the level of population over time, breaking up time into one-half year intervals. In this case, the first time at which population must be calculated is half-way through the year 2020. The population half-way through the year 2020 is just the population at the beginning of 2020, plus the number of babies born during the half-year interval. And, since 100 babies are born each year, one-half a hundred, or fifty are born in a half-year.

POP(2020.5) = POP(2020) + 0.5*100 = 5510 + 50 = 5560

By the same token, the population at the beginning of year 2021 can be calculated on the basis of the population in the middle of year 2020; the population in the middle of year 2021 can be calculated on the basis of the population at the beginning of year 2021; and so forth.

This procedure suggests a way to write a general equation that can be used to calculate the population at any moment in time, based on the population one time interval earlier. To clarify the development of the equation, it is helpful to refer to the moment in time at which the population is currently being calculated as the "present time," and it is helpful to refer to the interval between calculations as "one time interval."

The equation to be developed combines two fundamental ideas. First, the population at the present time (i.e., the time currently being calculated) equals the population one time interval earlier, plus the births that occurred over the interval. Second, the number of births occurring over one time interval equals the length of the interval, multiplied by the number of births per year. Combining these two ideas produces the following equation:

POP(present time) = POP(one time interval earlier)
+ (length of time interval)*BIRTHS(per year)

As equations go, this one appears somewhat cumbersome. One way to improve matters is to use symbols for the terms "present time," "one time interval earlier," and "length of time interval." Although many symbols are possible, the following are used throughout the text because they are consistent with the notation used by the DYNAMO simulation language to be introduced in Chapter Fourteen.

LEVEL.K	a level calculated at the present time
LEVEL.J	a level calculated one time interval earlier
DT	the length of the time interval between J and K

Figure 13.12 displays these symbols in graphic form. (The symbol L appearing in the figure will be discussed later in the chapter.)

Using this notation, POP(present time) is written POP.K, and POP(one time interval earlier) is written POP.J. Thus the level equation for population can be written:

POP.K = POP.J + DT*BIRTHS

For clarity, it is conventional to express the product of DT and BIRTHS as (DT)(BIRTHS). Hence the equation for population would generally be written:

POP.K = POP.J + (DT)(BIRTHS)

This equation can be read, "The population at time K equals the population at time J plus DT multiplied by BIRTHS."

Altogether, our model of the Xanadu population includes two equations. The first is a simple rate equation, indicating that the number of births per year is 100. The second equation is a level equation, indicating that the change in population over one time interval equals the number of births per year times the length of the time interval.

BIRTHS = 100
POP.K = POP.J + (DT)(BIRTHS)

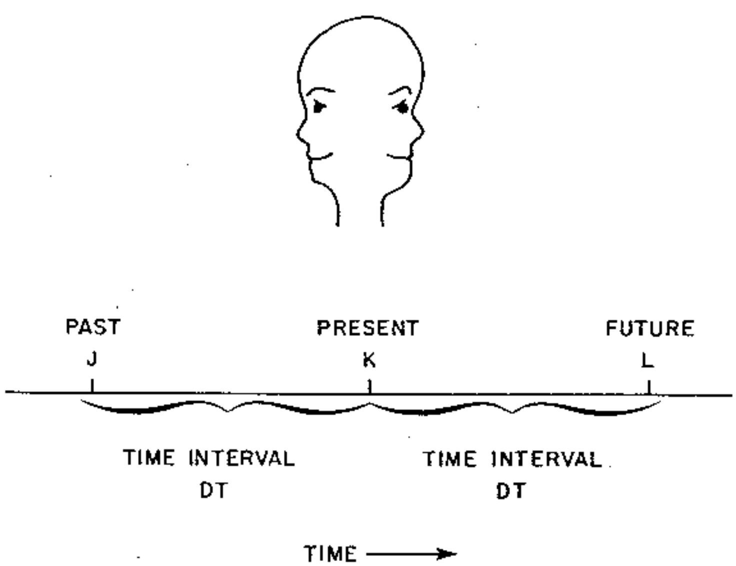


Figure 13.12 Definition of timescripts

In general, to simulate a model, it is necessary to write an equation for each level and each rate in the model, much as in the Xanadu case. The Xanadu model is a bit unusual in one respect, in that the rate (the number of births per year) is a constant. Often, rate equations are more difficult to formulate. But level equations are generally formulated exactly as the level of population in Xanadu. The value of a level at the present time *must* equal its value one time interval earlier, plus whatever flowed into the level over the time interval (minus whatever flowed out).

Consider, for example, the parking lot illustration discussed at the beginning of the chapter. (See Figure 13.1.) According to the example, the number of cars in the parking lot is a level, and the number of cars arriving per hour is a rate. If the name CARS is used to represent the number of cars in the lot, and ARRIV is used to represent the arrival rate (in cars per hour), then the level equation for CARS can be written:

CARS.K = CARS.J + (DT)(ARRIV)

The flu example shown in Figure 13.2 provides a somewhat more complex illustration. The number of children sick is a level, and the number of children who become infected per day, as well as the number who recover per day, are rates. If the name INFEC is used to represent the infection rate (in children per day), RECOV is used to represent the recovery rate (in children per day), and NSICK is used to represent the number of children sick, then the level equation can be written:

NSICK.K = NSICK.J + (DT)(INFEC-RECOV)

Exercise 5: Writing Level Equations

Review Exercise 1, and then write *level* equations for each of the levels in parts (a) through (d). (Choose whatever variable names you wish, and write them in ALL CAPS. Try to pick names that will aid you in remembering the subject of the equation!)

EXAMPLE III. CALCULATING THE POPULATION OF XANADU

The equations for POP and BIRTHS developed in Example II can be used to hand-simulate the population of Xanadu over time. As the year-by-year calculations are carried out, it is convenient to record the results in a form similar to Table 13.1.

Table 13.1 Table for computing population of Xanadu

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	100
2021	100	5610	100
2022	100	5710	100
2023	100	5810	100
2024	100	5910	100
2025	100	6010	100
2026	100	6110	100
2027	100	6210	100
2028	100	6310	100
2029	100	6410	100
2030	100	6510	100
2031			
2032			
2033			
2034			
2035			
2036			
2037			
2038			
2039			
2040			

Population in 2020 = 5510 people
Births = 100 people/year
Time Interval = 1 year

Year 2020. At the beginning of the simulation, the present time or time K is the year 2020. To get the simulation going, an "initial value" for the population in 2020 must be selected. For Xanadu, the population is known to be 5510 people in the year 2020. Thus, 5510 is entered in the *Population* column of Table 13.1, for the year 2020. The table then looks as follows:

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	

The next thing that must be calculated is the birth rate for the year 2020. In this case, births are calculated according to the equation

BIRTHS = 100

Thus 100 is entered under *Births* for the year 2020, yielding an entry like this:

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	100

Year 2021. At this point, the calculations for the year 2020 are complete. To carry out the calculations for the year 2021, it is necessary to "advance the calendar" one year. Thus the year 2021 becomes the "present time" (time K) and the year 2020 becomes time J. The population in the year 2021 can then be calculated using the formula:

POP.K = POP.J + (DT)(BIRTHS)

The calculation is easiest if (DT)(BIRTHS) is computed first and written down. The column *Change in population* in Table 13.1 is reserved for this purpose.

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	100
2021	100		

The population in the year 2021 (time K) can then be computed by adding the population in year 2020 (time J) to the *Change in population* column.

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	100
2021	100	5610	

The birth rate for the year 2021 can then be calculated as before, using the rate equation BIRTHS = 100.

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	100
2021	100	5610	100

Year 2022. The calculations for the year 2021 are now complete, and, once again, it is necessary to advance the calendar another year. Thus the year 2022 becomes the "present time" (time K), and the year 2021 becomes time J. Then, computations can be carried out exactly as in the year 2021, producing the following results.

Time	*Change in population*	*Population*	*Births*
2020	-----	5510	100
2021	100	5610	100
2022	100	5710	100

The simulation can be continued for as long as needed, with each advance of the calendar producing another iteration.

Exercise 6: Computing Additional Values for Population

Following the procedure in Example III, compute the population of Xanadu through the year 2040. Graph the results.

Exercise 7: Population at Other Times

If the population of Xanadu is 5510 at the beginning of year 2020, what is the population after the first month of 2020? What is the population in the year

2520? Is there a way to compute the answer without iteratively calculating the numbers as in Example III?

EXAMPLE IV: WORLD POPULATION GROWTH

Much the same approach used in writing the equations for the Xanadu model can be used in writing equations for a model of the growth of world population. Consider the causal loop and flow diagram in Figure 13.13. According to the diagram, the number of net births each year is influenced by the size of the population, and the size of the population is influenced by the number of net births. (The number of net births each year is the difference between the number of births and the number of deaths.)

The main problem involved in formulating a model of world population growth is formulating the rate equation for net births. In Xanadu, the rate equation was simple, since the number of births each year was constant, but for the world population, the number of net births each year is not constant. It increases as the size of the population increases.

What sort of equation should be written to express the relationship between the size of the world population and the number of net births each year? Many alternative formulations are possible, but the simplest assumption is that the number of net births each year is a *constant percentage* of the world population. In fact, over the recent past, the world population has grown at about 2 percent per year. This means that 0.02 net births are generated each year for every member of the population. Or, perhaps more sensibly, two net

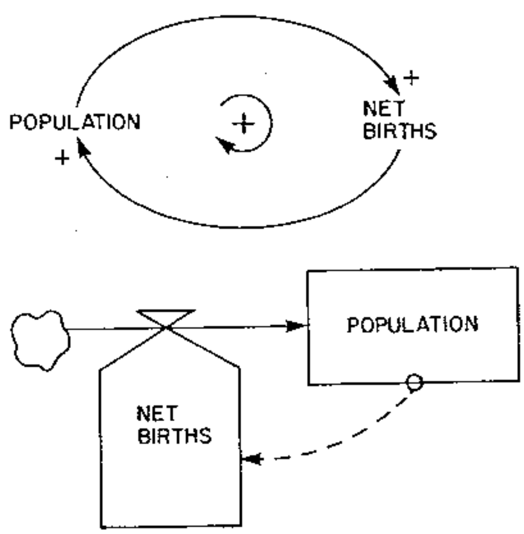


Figure 13.13 Causal-loop and flow diagrams for world population

births are generated each year for every 100 members of the population. (These 2 net births might correspond to a combination of 3 deaths and 5 new babies born.)

In analyzing population growth, it is essential to distinguish between the net birth rate, measured in *people per year,* and the annual percentage growth in the population, measured in *percent per year.* To call attention to this distinction, call the percentage growth in the population the growth fraction, or GF.

Using the growth fraction GF, the rate equation for net births NBIRTH (in people per year) can be written:

NBIRTH = POP.K*GF

Precisely speaking, the growth fraction GF equals 0.02, and it is measured in units (persons/year)/person. That is, 0.02 persons per year are added to the population, for each person in the population. The expression (persons/year)/person can be reduced algebraically to the expression (1/year), which in words is simply "per year." The expression (1/year) may seem odd at first glance, but after some reflection, it should be clear that a percentage growth of 2 percent per year amounts to a growth fraction GF = 0.02 "per year."

Once the rate equation for net births has been formulated, the level equation for population can be written rather easily. It has the usual form:

POP.K = POP.J + (DT)(NBIRTH)

Thus the complete model for world population growth consists of two equations:

NBIRTH = POP.K*GF
POP.K = POP.J + (DT)(NBIRTH)

One small technical matter needs to be taken care of. The equation for net births indicates that the number of births per year depends on the size of the population—and, of course, the size of the population varies over time. In the equation for net births, the variable POP has a subscript K to indicate the time, but net births NBIRTH so far does not. What subscript should be used?

The easiest way to determine the answer is to carry out a hand-simulation. For simplicity, carry out the simulation using a time interval DT equal to one year, and begin the simulation in 1975, when the world population was roughly 4 billion. According to the rate equation for net births, the number of net births per year over the period 1975 to 1976 is:

NBIRTH = POP.K*GF
= 4*(0.02)
= 0.08 billion persons per year

Furthermore, the size of the population in 1976 is simply the size in 1975, plus the number of net births that occurred over the year interval from 1975 to 1976.

$$\begin{aligned}\text{POP.K} &= \text{POP.J} + (\text{DT})(\text{NBIRTH})\\ &= 4 + (1)*(0.08)\\ &= 4.08 \quad \text{billion persons}\end{aligned}$$

Since the net birth rate used in the calculation of the population in 1976 is, by definition, the birth rate that persists over the interval 1975 to 1976, it is plausible to give the net birth rate two subscripts: one for 1975 and one for 1976. In assigning these subscripts, however, it is necessary to pay strict attention to the "calendar time" at which the calculations occur. The net birth rate for the period 1975 to 1976 was calculated on the basis of the population in 1975, when the "present time" (time K) was 1975. Thus the rate equation should be written:

$$\text{NBIRTH.KL} = \text{POP.K}*\text{GF}$$

This indicates that the net births per year during the period from time K (1975) through time L (1976) is equal to the population in 1975, times the growth fraction GF. (Recall from Figure 13.12 that time L is one time interval DT following time K.)

The population in 1976 is calculated when the "present time" is 1976. Thus the level equation for population should be written

$$\text{POP.K} = \text{POP.J} + (\text{DT})(\text{NBIRTH.JK})$$

This indicates that the population in 1976 (time K) equals the population in 1975 (time J) plus the number of net births between 1975 and 1976.

Taken together, then, the full model of the world population should be written:

$$\begin{aligned}&\text{POP.K} = \text{POP.J} + (\text{DT})(\text{NBIRTH.JK})\\ &\text{NBIRTH.KL} = \text{POP.K}*\text{GF}\end{aligned}$$

The detailed steps involved in simulating world population can be carried out most easily by constructing a table similar to the table used in the Xanadu example. Table 13.2 depicts a table for computing world population.

Year 1975. When the simulation begins, the present time (time K) is 1975. The initial value of the world population (4 billion) is entered as the value of population in 1975, and then the net birth rate for the interval 1975 to 1976 can be calculated according to the equation:

$$\text{NBIRTH.KL} = \text{POP.K}*\text{GF}$$

Table 13.2 Table for computing world population

Time (years)	*Change in population (people)*	*Population (people)*	*Net births (people/year)*
1975	-----	4.00	0.08
1976	0.08	4.08	0.08
1977	0.08	4.16	0.08
1978	0.08	4.24	0.08
1979	0.08	4.32	0.09
1980	0.09	4.41	0.09
1981	0.09	4.50	0.09
1982	0.09	4.59	0.09
1983	0.09	4.68	0.09
1984	0.09	4.77	0.10
1985	0.10	4.87	0.10
1986			
1987			
1988			
1989			
1990			
1991			
1992			
1993			
1994			
1995			
1996			
1997			
1998			
1999			
2000			

The resulting net birth rate (0.08 billion persons per year) is then entered in the table, producing the following results.

Time	*Change in population*	*Population*	*Net Births*
1975	-----	4.00	0.08

Year 1976. At this point, the calculations for 1975 are complete, and the calendar is advanced one year. Thus the present time (time K) is now 1976. The year 1975 has become time J, and 1977 is time L. The population in 1976 can now be calculated, using the level equation:

POP.K = POP.J + (DT)(NBIRTH.JK)

The value used for net birth is, of course, the value for the period 1975 to 1976, which is the value calculated as the final computation of the 1975 simulated year. To calculate the value of population in 1976, the product (DT)(NBIRTH.JK) should be entered in the table under the column *Change in population,* and then the product can be added to the population in 1975.

Once the population for 1976 is calculated, net births for the period 1976 to 1977 can be computed, using the rate equation:

NBIRTH.KL = POP.K*GF

This produces the following results.

Time	*Change in population*	*Population*	*Net Births*
1975	-----	4.00	0.08
1976	0.08	4.08	0.08

These steps can then be continued for as many iterations as are desired.

Exercise 8. Computing World Population

Following the procedure outlined in Example IV, compute the world population through the year 2000. Graph the results.

ENDNOTE

1. This accounts for the occasional awkwardness in reading a causal-loop diagram, discussed in Chapter 3.