

# Thermodynamics and economics

Alastair D. Jenkins

*Bjerknes Centre for Climate Research, Geophysical Institute, Allégaten 55, N-5007  
Bergen, Norway*

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## Abstract

The application of principles of thermodynamics and statistical mechanics to economic systems is considered in a broad historical perspective, extending from pre-historic times to the present day. The hypothesis of maximum entropy production (MEP), which has been used to model complex physical systems such as fluid turbulence and the climate of the Earth and other planets, may be applied to human economic activity, subject to constraints such as the availability of suitable technology, and the nature of political control. Applied to the current abundance of available energy from fossil fuel reserves, MEP is shown to have significant policy implications.

*Key words:* Economic systems; Statistical mechanics; Thermodynamics; Maximum entropy production principle; Energy supply; Political control

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## 1 Introduction

In this paper I discuss the applicability of concepts and techniques of thermodynamics and statistical mechanics to areas within social science and economics. In particular, the concepts of entropy, and of entropy *production*, are shown to be important. Although the production of entropy, in its information-theoretic sense of randomness, has been shown to be a driving factor in the effectiveness of markets (Maasoumi and Racine, 2002), on the macro-economic scale of whole societies it appears that a tendency for the maximisation of the production of *classical thermodynamical entropy*, via the consumption of available energy and other resources, may play a dominant role. The nature of economic and social development and evolution will thus be determined by

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*Email address:* [alastair.jenkins@bjerknes.uib.no](mailto:alastair.jenkins@bjerknes.uib.no) (Alastair D. Jenkins).

*URL:* <http://www.gfi.uib.no/~jenkins/> (Alastair D. Jenkins).

the physical and other constraints which are placed on entropy production and energy consumption. The efficiency of systems of economic and political control will depend on how well they take into account these underlying driving forces. The application of the principles of thermodynamics and information theory in economic systems is described by Ruth (1995; 1996; 2005), who states that ‘concepts and measures available from physics can be used to improve our understanding of economic evolution if properly placed into the context of socioeconomic processes’. In the present paper we consider the thermodynamic and statistical mechanical aspects of economic processes, also from a historical perspective.

## 2 Physical Background

If we try to obtain complete solutions of the equations governing the mechanics of a system with many degrees of freedom, we are faced, in general, with an intractable problem. A system with as few as three degrees of freedom may be subject to chaotic behaviour, in which a small perturbation in the initial conditions will grow exponentially, so that the detailed long-term behaviour is essentially unpredictable (Lorenz, 1963; Klavetter, 1989). This intractability is an essential feature of macroscopic systems which are composed of many atoms and molecules, one of the simplest type of which, *ideal gases*<sup>1</sup>, being the subject of intensive study during the 18th and 19th centuries. The study of ideal gases culminated in the kinetic theory of Maxwell (1890) and the statistical mechanics of Boltzmann (1872; 1877) and Gibbs (1902).

The basis of the kinetic theory of gases is that macroscopic properties such as temperature and pressure are computable even though the motions of the individual gas molecules may only be specified in statistical terms. Indeed, Maxwell (1890) and Boltzmann (1872) showed that the components of the molecular velocity along any coordinate direction are normally distributed with a variance proportional to temperature and inversely proportional to the mass of the molecule, the constant of proportionality  $k$  being equal to what is now termed *Boltzmann’s constant*.

That gases have elegant mathematical properties was well known before the development of kinetic theory. In the mid-17th century, Robert Boyle found that for a fixed mass of gas, the pressure and volume at a given temperature were inversely proportional to each other. Subsequent work by Charles, Gay-

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<sup>1</sup> Gases whose molecules effectively occupy only a negligibly small fraction of the space they move about in.

Lussac, and Avogadro, led to the formulation of the ideal gas law

$$pV = nRT,$$

where  $p$  is the pressure,  $V$  the volume of a sample of  $n$  moles<sup>2</sup> of gas,  $T$  a suitable temperature scale (now the absolute temperature in degrees Kelvin), and  $R$  a universal constant.

Parallel to the development of gas theory there developed the theory of heat engines. On the basis that heat cannot flow spontaneously from a colder to a warmer body, Sadi Carnot (1824) showed that a heat engine operating between two temperatures could not be more efficient than a reversible heat engine operating between the same temperatures, and that all reversible heat engines operating between two given temperatures have the same efficiency. This conclusion, together with Joule's demonstration of the equivalence of mechanical work and heat, led, via the work of Kelvin and Clausius (see Jaynes, 1988), to the statement of the laws of thermodynamics:

**[First law:]** Total energy is conserved: mechanical energy and heat are equivalent quantities;

**[Second law:]** There exists a state variable of a system (entropy). The entropy of an isolated system cannot decrease.

**[Third law:]** There exists a temperature (absolute zero,  $T = 0$ ) for which the entropy of any system tends to a constant value (which may often be taken to be zero) as it is approached.

The rather mystical quantity *entropy* is, nevertheless, quite well defined. For an ideal monatomic gas such as helium it is<sup>3</sup>

$$S = nR \left( \log \frac{V}{n} + \frac{3}{2} \log T + \text{constant} \right). \quad (1)$$

Useful thermodynamic relations to be aware of are:

$$\Delta E = Q + W,$$

which represents the First Law of thermodynamics,  $\Delta E$  being the change in total energy of the system due to a supply of heat  $Q$  and mechanical work  $W$ ;

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<sup>2</sup> A mole is now defined as the quantity of a substance with the same number of molecules as the number of atoms in 12 grammes of carbon isotope 12.

<sup>3</sup> This formula breaks down at low temperatures: the Third Law of thermodynamics predicts  $S \rightarrow 0$  rather than  $S \rightarrow -\infty$  as  $T \rightarrow 0$ . In reality, the gas would condense to become liquid or solid at low temperatures.

and

$$dE = TdS - PdV, \quad (2)$$

which is the *fundamental thermodynamic relation*, valid for both reversible and irreversible changes of state,  $dE$ ,  $dS$ , and  $dV$  representing infinitesimal changes in energy, entropy, and volume, respectively. To obtain finite changes, the relation (2) should be integrated between the initial and final states:

$$\int_{\text{initial}}^{\text{final}} dE = \int_{\text{initial}}^{\text{final}} TdS - \int_{\text{initial}}^{\text{final}} PdV.$$

We may also employ the following formula, due to Clausius:

$$dS \geq dQ/T, \quad (3)$$

with equality where the change in the state of the system is reversible. This relates the change in entropy of a system to the amount of heat supplied to it. We see that if the heat flows into or out of a system at a high temperature, it produces a smaller change in the entropy of the system than if it flows in or out at a low temperature.

It may also be useful to employ *thermodynamic potentials*, such as the *enthalpy*

$$H = E + PV, \quad dH = TdS + VdP,$$

when considering changes at constant pressure, or the *Gibbs free energy*

$$G = E - TS + PV, \quad dG = -SdT + VdP, \quad (4)$$

for changes at constant pressure and temperature (e.g. phase changes such as melting and evaporation).

## 2.1 Non-thermal entropy changes

The entropy of a system may also change (increase) in processes where there are no thermal effects, for example, during the mixing of two different gases which do not react with one another. Consider two adjacent compartments  $C_1$  and  $C_2$ , separated by a partition, each of volume  $V/2$ , containing  $n/2$  moles of monatomic gas species 1 and 2, respectively. By (1), the entropy of the gas

in each compartment is  $S_i = (n/2)R[\log(V/n) + (3/2)\log T + c_i]$ , where  $i$  is either 1 or 2, the  $c_i$  being the relevant constant in (1) for each gas. The total entropy is thus

$$S_{\text{initial}} = S_1 + S_2 = nR \left[ \log \frac{V}{n} + \frac{3}{2} \log T + \frac{1}{2}(c_1 + c_2) \right].$$

If we then open the partition between the compartments and let the gases mix, keeping them at the same temperature, each gas species will then occupy volume  $V$ , and the total entropy will become

$$S_{\text{final}} = S_1 + S_2 = nR \left[ \log \frac{2V}{n} + \frac{3}{2} \log T + \frac{1}{2}(c_1 + c_2) \right] = S_{\text{initial}} + nR \log 2.$$

This is a elementary example to show that entropy is a measure for the disorder of a system.

## 2.2 Relation between thermodynamics and microscopic dynamics

The relation between entropy and the microscopic dynamics of an ideal gas was clarified by Ludwig Boltzmann (1872), who showed that the following quantity would always decrease in a thermally and mechanically isolated system:

$$\mathbf{H} = N \int f(\mathbf{c}) \log f(\mathbf{c}) d\mathbf{c},$$

where the vector  $\mathbf{c}$  is the gas-molecule velocity, and  $f(\mathbf{c})$  is its probability density. The entropy per unit volume turns out to be  $-k\mathbf{H}$ , up to an additive constant. Subsequently, Boltzmann (1877) generalised this result to his famous formula

$$S = k \log W, \tag{5}$$

where  $W$  is a measure of the volume occupied by the dynamical system comprising the ideal gas molecules, in the phase space of  $6N$  dimensions, whose coordinates are the components of position and momentum of all  $N$  molecules.

The fact that  $\mathbf{H}$  always decreases appears to be paradoxical, as the equations describing the dynamics of the gas molecules and their interactions (elastic collisions) are time-reversible (Loschmidt's Paradox). In Boltzmann's lifetime, this was regarded as a serious defect in his theory. Boltzmann himself made the assumption of molecular chaos (*Stoßzahlansatz*), that is, that before the collision of any two gas molecules, the molecules' velocities are uncorrelated, and

it is from this assumption that we may derive a time-asymmetric equation for the evolution of the molecular velocity distribution (the Boltzmann equation) from the time-symmetric equations of dynamics. The validity of Boltzmann’s approach for practical purposes has been confirmed by the fact that it is able to predict, very accurately, using Enskog’s (1917) perturbative solution and related approaches (Chapman, 1916; Chapman and Cowling, 1970), many of the transport properties of gases, such as viscosity, thermal conductivity, diffusion coefficients, and also to predict quantitatively more non-intuitive effects such as thermal diffusivity (the tendency for a concentration gradient to be set up in a gas mixture when there is a temperature gradient). More recently, it has been shown that such apparently time-asymmetric behaviour can arise from time-symmetric equations used in simulations of molecular dynamics, as a result of the presence of both repelling and attracting subsets of the phase space (Nosé, 1984; Holian et al., 1987).

### 2.3 Mixtures of substances

The thermodynamics of mixtures of substances was put on a firm mathematical footing by J. Willard Gibbs (1875–1878), who introduced the concept of *chemical potential*  $\mu$ . For a system with  $N$  different substances, equation 2 is replaced by

$$dE = TdS - PdV + \sum_{i=1}^N \mu_i dn_i, \quad (6)$$

where  $n_i$  is the number of moles of substance  $i$ , which has a chemical potential  $\mu_i = (\partial G / \partial n_i)_{T,P}$ , the partial derivative of the Gibbs free energy (see Eq. 4). For substances participating in chemical reactions, chemical equilibrium is reached when the sum of chemical potentials of the reactants is the same as the sum of the chemical potentials of the products.

#### 2.3.1 Statistical mechanics of mixtures

Gibbs also laid the foundation for the statistical mechanics of general systems in thermodynamic equilibrium (Gibbs, 1902). His formulation was sufficiently general that it remained valid with the development a quarter-century later of quantum mechanics. The general principle behind Gibbs’ theory is essentially the maximum-entropy argument devised by Boltzmann, but applied to dynamical systems more general than those describing the behaviour of ideal gases, and with additional consideration of the presence of molecules of different chemical composition. From maximising the number of ways the phase space of the whole system may be occupied, subject to the system having its

given energy, momentum, and composition, Gibbs showed, in the limit of large particle number, that the behaviour of a system in thermodynamic equilibrium could be described in terms of what is now called the *grand partition function*

$$Z = \left\langle \exp \left[ \left( -E + \sum_{j=1}^m \mu_j n_j \right) / (RT) \right] \right\rangle,$$

where the angle brackets represent the average or mathematical expectation. Gibbs' theory, although at the time expressed in terms of the laws of classical mechanics, is also applicable to systems which obey the laws of quantum mechanics, developed in the 1920s by Heisenberg, Dirac, and Schrödinger, among others, where the positions and momenta of the components of the system must be expressed as abstract linear operators rather than as numbers.

## 2.4 Non-Equilibrium Systems

So far, we have only considered quantitatively the behaviour of systems in thermodynamic equilibrium. A mathematical theory of a non-equilibrium phenomenon—that of heat conduction—was developed by Fourier (1826). In microscopic terms, for a gas, the heat is carried and transmitted via the random molecular motions, the quantitative theory being developed by Maxwell and Boltzmann for a specific case of the intermolecular potential (proportional to the  $-5$  power of the distance between the molecules), and by Chapman and Enskog for general intermolecular force laws. Heat conduction in solid substances is via a similar mechanism, the role of the gas molecules being played by *phonons*, quantum-mechanical particles which are associated with elastic vibrations of the material.

A general quantitative theory of non-equilibrium thermodynamics was developed by Onsager (1931a,b). Onsager's theory is valid for systems which are close to thermodynamic equilibrium, in the sense that there is a linear relation between *thermodynamic forces* (such as the temperature gradient) and *flows*, such as the heat flux. In addition to heat conduction and molecular diffusion (in solids and liquids as well as gases), Onsager's theory covers phenomena such as the thermoelectric effect (heating or cooling caused by the passage of an electric current between two different materials), thermal diffusion (the diffusive separation of different substances in the presence of a temperature gradient), and so on.

For systems far from thermodynamic equilibrium, the general statistical mechanical theory has until recently been incomplete. It has been observed that there is often a general tendency for the rate of entropy production to be

maximised. For systems in a (stochastically) steady state which maintain a near-constant temperature, this is equivalent to saying that the rate of energy dissipation is maximised.

The hypothesis of maximum entropy production (MEP) has been applied with some success within the field of fluid mechanics, providing some quantitative results relating to the properties of the notoriously intractable problem of turbulent fluid flow. Busse (1970) and Malkus (1956) found some solutions to the hydrodynamic equations for a shear flow which maximised the viscous energy dissipation. Although these solutions, involving complex stationary flow patterns, were not strictly turbulent, they do give rates of cross-flow momentum flux (equivalent to turbulent shear stress) which are remarkably close to those observed in laboratory experiments and in fine-scale time-dependent numerical model simulations. A similar approach has also been applied with some success in the study of thermal convection.

On a more ambitious level, the MEP hypothesis has been applied to the even more complex system of the global climate (Paltridge, 1975, 1979; Ozawa and Ohmura, 1997; Pujol and Fort, 2002). This work has been applied to the ocean thermohaline circulation climate system (Shimokawa and Ozawa, 2002), and Lorenz *et al.* (2001) have shown how MEP may be applied to make quantitative predictions of the climate of other planetary bodies (Mars, Venus, Titan). An overview of the application of MEP to turbulence and climate-related studies is given by Ozawa *et al.* (2001).

Theoretical progress in understanding the concept of maximum entropy in non-equilibrium systems was made by Dewar (2003), who showed that MEP in a system in a steady state would be attained by the maximisation of *path information entropy*

$$S_I = - \sum_{\Gamma} p_{\Gamma} \log p_{\Gamma}, \quad (7)$$

where the summation index (or integration variable)  $\Gamma$  is over possible paths in the phase space of the dynamical system, and the  $p_{\Gamma}$  are the probabilities of the system will follow the individual paths. Equation 7 was first employed by Jaynes (1957), employing the information-theory entropy concept of Shannon (1948). In the case of systems in thermodynamic equilibrium, it reduces to the statistical-mechanical definitions of entropy deduced by Boltzmann (1877) and Gibbs (1902), but may also be applied to systems not in equilibrium. It should be noted that the paths  $\Gamma$  in (7) are restricted to those which are dynamically realisable: for example, a system constrained initially to be in a non-equilibrium state will tend to equilibrium with a finite relaxation time, given, for example, by the dynamics of intermolecular collisions. This principle of maximal  $S_I$  has been shown to reproduce the results of Onsager's theory,



and is also capable of making predictions of the behaviour of systems far from equilibrium (Jaynes, 1979; Robertson, 1966, 1967, 1993). A review of the application of the theory of dynamical systems to non-equilibrium statistical mechanics is given by Ruelle (1998).

The above principle of maximum of path entropy may be used to explain, in addition to processes involving heat flow, such phenomena as the behaviour of systems subject to critical behaviour, for example, a sand pile (Bak et al., 1987). The MEP property of networks exhibiting self-organised criticality has been applied by Lorenz (2003) to economic market systems, where the profit (difference between buying and selling price) realised by a participant in the market plays the part of energy dissipation or entropy production. Lorenz's idea can be thought of as a microeconomic application of MEP. On the macro-economic level of whole economies, I contend that MEP may be applied using the *usual thermodynamic definition* of entropy. To understand how this is so, I will outline human economic activity within its historical context.

### 3 Human Activity

The distinctive contribution of human activity to the global thermodynamic balance comes with the exploitation of fire. Before that time, human influence was effectively indistinguishable from that of other forms of living organism. Living systems tend to have a high degree of order (negative entropy), but they maintain this state by 'feeding' on energy and mass sources with a low specific entropy, and producing waste products with a higher entropy. For example, plants utilise radiant energy from the Sun, with an effective temperature of over 5000 K, and release the energy as heat at an ambient temperature of about 300 K. From Eqs 2 and 3, we see that the plants will tend to produce entropy. Similarly, the entropy of food which animals consume is less than the entropy of the waste products which they produce. It also appears that the appearance of vegetation on the planetary surface tends to reduce both the temperature and the albedo of the surface, thus increasing the entropy production rate for a given input of solar radiation (Ulanowicz and Hannon, 1987; Schneider and Kay, 1994; Kleidon et al., 2000; Kleidon and Lorenz, 2005).

Although naturally-occurring fires, caused, for example, by lightning strikes, have always existed, the generation and exploitation of fire by human activity has increased entropy production by enabling a more rapid dissipation of the free energy available from organic carbon and atmospheric oxygen, and the distribution of the effects of fire over a wide area, for example, to increase the availability of game for hunting (Beaton, 1982). Indeed, the supply of firewood has been a significant limitation on human economic activity, in historical periods, for example, in England in the fifteenth and sixteenth centuries (Lee,

2003), and in the present day in semi-arid mountain regions where the main economic activity is subsistence farming and grazing (Eckholm, 1975).

Entropy is a concept which, in addition to being a thermal property of materials, is also a property of the distribution of the materials. The entropy of a fixed mass of gas, for example, depends on the volume it occupies (see Eq. 1). The same applies for substances in solution (Debye and Hückel, 1923) or in mixtures. A prehistoric example of entropy production by this mechanism is given by the exploitation of flint resources—the production of tools, such as in the flint-mining areas of Norfolk, U.K. (Barber et al., 1999). A concentrated supply of flint is broken up and dispersed, as useful objects which are eventually discarded, and also as waste material.

The development of agricultural techniques enabled the increase by digging of the entropy of the soil and the release of mineral resources for plant growth, whose subsequent depletion coupled with an increased human population led to a rapid extension of the agricultural frontier, a process which took place in Europe, Asia, and Africa in prehistoric times, and which was repeated in the Americas and Australasia in more recent historical periods. The expansion of agriculture is an early example of available free energy contributing both to direct entropy production and by ‘investment’ in ‘entropy-productive capacity’ in the form of forest clearance and other activities which enable future increase in entropy production.

Agricultural techniques enable a greater production of ‘fuel’, for humans, livestock, and ‘pests’, than would otherwise be possible. The necessary increased absorption of sunlight for photosynthesis will decrease the Earth’s albedo correspondingly, increasing the energy absorbed (and thus the entropy produced) from solar radiation (Kleidon et al., 2000). This will be the case both for when the original landscape is forested, and when the original landscape is arid and subsequently irrigated for food production.

The production of tools by smelting of metal ores (copper, tin, iron), although it produces end products of lower entropy than the original raw materials, will necessarily produce entropy in the waste products. The tools produced will increase entropy production via the production of food, and their use in human conflicts will increase entropy in a more ‘disorderly’ manner. Eventually the tools will corrode, into substances of a similar specific entropy to the original ores, but their distribution will be less concentrated and thus have greater entropy.

Societies based on agriculture may of course reach considerable complexity and sophistication. The ‘entropy production’ of an agricultural society at a state in which the population is more-or-less constant (Laslett, 1979) may be maximised by the export of food and excess population to urban areas (which

have higher death rates). Social stratification may enable a more ‘effective’ entropy production. Laslett also states (1979, p. 66) that the controlling *gentry* in 17th century England ‘pressed, like the atmosphere, evenly, over the whole face of England’, that is, as in a body of gas evenly distributed throughout a container, their spatial distribution was in a state of maximal entropy. Conflicts with other societies provide an additional source of energy dissipation / entropy production and absorption of ‘excess population’.

## 4 Industrial Societies

Of course, the so far greatest expansion of anthropogenic entropy production, to an extent which will almost undoubtedly affect the global climatic balance, has occurred after the start of the Industrial Revolution. Large-scale iron production was previously limited by the timber available for charcoal-making, for example in the Caledonian Forest of Scotland (Tittensor, 1970; Dye et al., 2001). The scene was then set for the conversion of the huge geological reserves of coal and, later, petroleum, laid down over the past hundreds of millions of years, to atmospheric carbon dioxide ( $\text{CO}_2$ ), at an unprecedented speed. The fact that this process could not happen instantaneously is due to the energy investment necessary for coal mining, ore extraction, smelting works, transport infrastructure (such as railways) and other facilities and processes necessary for society to absorb the energy and materials generated. The large investment of energy required to construct and develop mines and processing resources limits the rate of increase of total production and consumption of coal. It can probably be shown that the coal production during the Industrial Revolution, in Britain, for example, increased at a rate which maximised the total entropy production, subject to the technological constraints which applied at the time. For example, we may assume that if the rate of coal extraction (in units of the energy available by burning the coal) is  $P$ , the energy used in coal extraction and distribution is  $\alpha P$ , and the power used to increase production capacity is  $hP$ . Furthermore, we assume the following relation between the rate of change of coal production and the investment in increasing production capacity:

$$\frac{dP}{dt} = \beta h(2h_0 - h)P - hP. \quad (8)$$

The rate of entropy production associated with the burning of  $Q$  units of coal is  $Q/T_a$ , where  $T_a$  is the ambient temperature. Under the assumption that  $T_a$  is constant, the entropy production will be maximised for  $h = h_0 - 1/(2\beta)$ , and the rate of coal extraction will increase exponentially<sup>4</sup>.

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<sup>4</sup> It may be argued that a higher rate of entropy production would be attained just by burning coal ‘in place’ instead of distributing it for ‘productive’ use. However,

The development of efficient transport systems such as railways tended to reduce the entropy production during the transport process itself. However, by increasing the total transport capacity and making the wider distribution of products possible, the total entropy production rate of the whole society and economic system would be increased, that is, economic growth would be stimulated. The effectiveness of the railways in the economic development of the U.S.A. in the 19th century was nevertheless disputed by Fogel (1964), who claimed that a combination of water and road transport would have been almost as effective. However, his conclusion was disputed by Holmes and Schmitz (2001), who pointed out that if the waterways had not faced rail competition, they would have been subject to a greater extent of restrictive practices, such as a tendency for individual groups, such as dockers' unions, to maximise their own entropy production (see also Rutten, 2003).

Passing on to present-day society, oil and gas are supplanting coal as they require the dissipation of less energy to produce and transport—thus their actual production (and the consequent entropy produced as they are consumed) may be greater than if coal was still used as fuel. Coal production thus tends to be concentrated in the reserves which are least energy-intensive to extract and transport to the markets, the other less 'economic' mines facing closure.

Today's abundant sources of fossil-fuel energy result in a situation whereby the tendency to maximum entropy production becomes highly visible. Transport provides a good example. Although for heavy bulk cargo the greater energy efficiency of marine and rail transport still gives an advantage to these modes, the tendency for MEP leads to the dominance of the modes with higher energy dissipation: road, and, increasingly, air transport. That rail has, in some countries, not entirely disappeared as a mode of medium-to-long-distance passenger transport, lies perhaps in the fact that the generally greater comfort of rail as opposed to bus transport leads to rail being comparably or even less energy-efficient per passenger-kilometre (Andersen et al., 1999; Andersen, 2001).

In metropolitan areas, there is a tendency towards the use of rail for passenger transportation, as it has a greater carrying capacity than road transport, and thus allows for higher levels of economic activity (and entropy production). Cars standing in traffic jams, although they may emit considerable amounts of noxious substances, consume relatively little fuel and thus make a smaller contribution to entropy production.

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not only is the human socioeconomic system not designed for such an 'unprofitable' activity (although a certain amount of spontaneous combustion does, in practice, unavoidably take place), but also if the coal is distributed to the wider economy it will enhance the consumption of other raw materials, and thus produce extra entropy at a rate proportional to the product of their chemical potential and their consumption rate (see Eq. 6).

It may be thought that the command economies of the former Soviet bloc countries provide a counterexample to the tendency for MEP. These economies, with their emphasis on coal/steel-based heavy industry and the consequently large energy consumption and entropy production, are being superseded by free-market economies which have somewhat lower energy consumption. However, we can consider this phenomenon as a result of the relaxation of a constraint—the entropy production in the part of the society outside the industrial complex was less than it would have been in the absence of the rigid system of political control. Indeed, the economies of the former Eastern bloc countries are now showing MEP characteristics of free-market economies, such as reliance on private motor vehicles, rather than less energy-intensive public transportation (Hanell et al., 2000).

Modern agriculture has increased its entropy production above what could be achieved in the pre-industrial age. The availability of cheap extractive and transportation technology has enabled the transport of minerals such as phosphate to agricultural areas. The fixation of nitrogen from the air to make it available for fertilising plant growth may be performed industrially, via chemical engineering technology such as the Haber process for producing ammonia (Smil, 2004), thus short-circuiting the slower naturally-occurring bacterial processes.

The heavy investment required for industrial development has given rise to a tendency for ‘business cycles’, a phenomenon recognised from Victorian times (Duesenberry, 1958). The price depression caused by increased capacity and competition then leads to reduced profits for investors. Political forces may then arise, with the aim of maintaining employment, leading to a tendency for the provision of subsidies, or the imposition of tariff barriers to restrict competition. This political process can be thought of as a means to maintain the entropy production of the industrial process concerned, although the wealth (money) production in the economy as a whole will not be optimal.

The recent ‘dotcom bubble’ may also be interpreted in terms of entropy production. The amount of bits of information produced in the information technology industry is vast, but its ‘Shannon entropy’ is negligible compared with the associated entropy production in industrial production, transportation, building, and so on. Even though many investors lost a lot of money, the industry they invested in generated considerable entropy.

There has been recent discussion of the benefits of free and open source computer software, which may be most efficient in the long run, since there is a greater potential to build new and more useful software products in the absence of restrictions on sharing computer source code (Mustonen, 2003). At present, however, there is much more money (and thus entropy) produced in the market for traditionally-produced commercial software products. The

same condition applies to other forms of intellectual property, such as books, music, and so on. The owners of copyrights and patents produce entropy (for themselves) ‘while the sun shines’.

## 5 Environmental Implications

Tendency for maximisation of entropy production leads to rapid use of available resources, and a general tendency for increased environmental pollution. The rapid use of fossil fuel reserves, built up over hundreds of millions of years, may very well produce a serious greenhouse-gas-induced climate change before their eventual exhaustion. It will be a challenge for the world’s political system to impose suitable constraints, particularly since the potential for entropy production from renewable resources such as solar and wind energy is at first sight quite restricted.

What appear to be far-fetched ideas for storing waste  $\text{CO}_2$  in subterranean storage reservoirs, such as depleted oil and gas fields, may in fact be realistic from an entropy-production viewpoint. A substantial proportion of the energy produced will need to be consumed in compression and transport of  $\text{CO}_2$  to the reservoirs, but this, of course, will lead to an increase in entropy production, which may be favoured by economic and political forces.

## 6 Implications for Economic Planning

We have seen in the previous sections how human activity, as well as the Earth’s geophysical and biological processes, may act in a way to produce entropy at a maximum rate, subject to practical constraints such as the availability of resources and other raw materials, and the energy which must be diverted in order to extract them. It is consistent with the idea that economic growth will be maximised if the economic system is subjected to as little disturbance as practicable: rigid economic planning is liable to reduce the rate of growth, even if the plans do not intentionally have this effect. Another implication of the MEP hypothesis is to suggest an alternative, perhaps more objective, alternative to the concept of economic *utility*. Instead of individuals acting to maximise their own, highly subjective ‘utility’, which is expressed in the market in monetary terms, we may hypothesise that individual and political pressures expressed in society as a whole tend to maximise entropy production. This again suggests further how political power in a society is not merely dependent on the financial resources available to individuals or corporations, but is alternatively a function of the amount of energy and material resources they control for entropy production. We would therefore expect that

corporations which control access to energy reserves, and also governments which regulate the allocation of such reserves, should exert more political and economic power, even if in financial difficulty, than those without such access and control. The deliberate addition to motor gasoline of the toxic chemical tetraethyl lead, and the consequent astronomical increase of lead deposition throughout the global environment, is a case in point (Milberg et al., 1980).

It may thus be seen that the tendency for MEP due to human economic activity may not in all respects have beneficial effects. However, this MEP tendency is subject to the constraints which are either originally present or which may be imposed on the system. The economic systems in different parts of the world may act as a guide. The U.S.A. has the world's greatest entropy production *per capita*, and this is associated with a relatively unconstrained economic system. However, even in the U.S.A., uncontrolled economic activity is regulated by such means as local building codes, and national social security and environmental protection schemes. In Europe, the rather lower per capita entropy production is perhaps associated with a higher level of central infrastructure planning and social welfare levels, and manifests itself by means of a rather smaller spread of individual income, a somewhat greater use of public transportation, *et cetera*.

However, economic regulation may be driven by political pressures which act to increase entropy production. The high social welfare and public infrastructure spending in the Nordic countries (Dowrick, 1996) may be a response to those countries' generally low population density: entropy production may be maximised by keeping the population spread throughout the country rather than yielding to a natural centralising tendency. The agricultural subsidies applied in the European Union (de Gorter and Meilke, 1989) and elsewhere will also have a similar effect.

The Montreal protocol (Murdoch and Sandler, 1997) on the production of chlorofluorocarbons (CFC) and other substances that deplete the stratospheric ozone layer can also be seen to be a MEP-driven agreement. When it was eventually realised that stratospheric ozone levels were being reduced significantly by CFC-catalysed chemical reactions in the atmosphere, and that the resulting levels of solar ultraviolet radiation were likely to increase, leading to an increased danger of skin cancer and possible adverse effects on oceanic plankton populations, the consequences to the tourist and fishery industries became clear. The possible reduction in economic activity (entropy production) in these industries helped to generate the political pressure which led to a successful agreement. The proposed reduction in CO<sub>2</sub> releases envisaged in the Kyoto Protocol (Babiker et al., 2002) can then be seen to be much more difficult to attain, as they will lead to a *reduction* in global entropy production. If air transport is included in the CO<sub>2</sub> emission reduction requirements, this will obviously be vigorously resisted by the tourist industry. In the medium

term, an entropy production increase may be achieved by sequestration of CO<sub>2</sub> emissions, in geological formations, the ocean (Drange and Haugan, 1992; Brewer et al., 1999), or elsewhere, a process which will require a significant fraction of the global total energy production. This, however, will require a large investment in processing capacity, in energy as well as in financial terms.

Model studies of the changes in industrial processes induced by different types of carbon emission reduction incentive have been performed by Ruth et al. (2000) and Ruth and Amato (2002) for U.S. iron and steel production, and by Ruth et al. (2002) for U.S. ethylene production. In these studies, it was found that the response of the industries to different types of incentive varied: a CO<sub>2</sub> tax was not so effective in reducing emissions as policies, such as research and development stimuli, which were specifically directed at the industries concerned. The latter type of policy may be regarded as ‘catalytic’ in that it affects the ‘reaction kinetics’ or dynamics rather than affecting the state of economic equilibrium to be attained. In general, to influence industries and economic processes to direct them towards specific environmental goals, it is necessary to implement policies which both encourage a transition towards the specified goal, which must also be ‘thermodynamically’ realistic, and also ease the constraints which may inhibit such a transition.

It can thus be seen that the long-term future of human society, as affected by greenhouse gas induced climate change, provides a serious political challenge to the process of economic and social planning. The consequences of any imposed technical or economic regulations which purport to protect the global or local climate and environment should be assessed realistically, not only with regard to their ecological consequences and financial implications for the stake-holders, but also with regard to their implications for energy production and cycling and for entropy production. The entropy production factor may be used as a proxy for as-yet-unforeseen economic and political forces.

## **7 Concluding Remarks**

In this essay we have introduced concepts from thermodynamics and statistical mechanics, for both equilibrium and non-equilibrium systems, which may have applicability for the global economy and society generally. The system which contains the Earth’s climate, biosphere, human society and economy, is not at equilibrium, but is an open system, characterised by an energy flux from solar radiation which is re-radiated to outer space. Human activity, in addition, produces more energy from fossil fuels, which is also re-radiated, and also extracts minerals from ores and re-distributes their products.

Thermodynamic systems which are isolated have a tendency to reach equi-



librium, a state of maximum entropy, subject to constraints such as the total energy and the chemical composition. For open systems, at least those which are on average in a steady state, there has been observed a tendency for maximisation of entropy *production* (MEP), a hypothesis which has been proved mathematically, under certain circumstances, by Dewar (2003). This MEP tendency has been observed in the dynamics of the climate of the Earth and other planets, in fluid turbulence and convection, in the dynamics of network systems subject to self-organised criticality, and in the climatic effects of the biosphere, which suggests a ‘rational’ explanation of the ‘Gaia’ hypothesis that the Earth’s climate–biosphere system is self-regulating (Lovelock and Margulis, 1974).

It should be noted that alternative, related hypotheses for the behaviour of complex mechanical, thermodynamic, biological and economic systems have been put forward. One example is that of maximum utilisation of *available energy* or *exergy* (Edgerton, 1982; Kay, 1984; Schneider and Kay, 1994; ?; Fraser and Kay, 2002). The quantity ‘exergy’ may be defined as the maximum energy available to a system by the operation of an ideal heat engine which is able to exchange heat at the ambient environmental temperature. Exergy is said to have simpler conservation laws than (non-conserved) entropy, and to be easier to compute. Unlike entropy, the computation of exergy relies on the presence of an ambient heat bath. The concept of maximising entropy production should thus be of more general applicability than that of maximising exergy utilisation, although the two concepts will be equivalent under suitable circumstances.

In this paper I assume that MEP applies to human economic activity, the ‘entropy’ being the usual thermodynamic entropy. This has the consequence that macro-economic processes are directly related to available resources of energy, and also of minerals and other raw materials. In today’s society, with abundant energy available from fossil fuel reserves, economic activities, for example, modes of transport, are favoured if they consume more energy, and thus produce more entropy, than their alternatives. International agreements, even those which purport to restrict economic activities, such as the Montreal protocol (Murdoch and Sandler, 1997), will nevertheless be favoured politically if they will lead to increased entropy production, or, at least, will avoid the risk of reduced entropy production.

The major challenge facing world society today is that of global climate change induced by the emission of greenhouse gases such as CO<sub>2</sub>. Efforts, such as the Kyoto protocol, to limit or reduce greenhouse gas emissions, face severe political difficulties, since they will result in limits to entropy production. In order for such agreements to succeed, it must become clear, to each individual as well as to the political, intellectual, and commercial élite, that severe economic and social consequences will ensue, to the extent that even the foreseen

‘entropy production’ (in the way of, for example, social interaction, tourism, sport, and entertainment) will *decrease*, if an agreement to limit emissions is not made. In order for everyone to come to such a realisation, a substantial investment must be made, in the monitoring of climatic variables in order to detect early signs of adverse climate change, in research, both for understanding the present and past climate and climate–biosphere interactions, and for modelling and predicting future climate change and its effects. Finally, it will be necessary to put into place a rational, believable, and well-thought-out campaign of publicity and education, so that there is a general understanding of the prospects for climate change and its consequences. This, one may think, would be an overwhelming task, but the success of such (substantial entropy producing!) organisations such as *Greenpeace*<sup>5</sup> shows that it is possible to mount a successful challenge to entrenched economic interests.

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<sup>5</sup> *Greenpeace* gives information on its (entropy-producing) ships at URL <http://archive.greenpeace.org/ships.shtml>.

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