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From linearity to complexity: Towards a new economics

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Abstract

In this paper we analyze the changes recently undergone in pure research and applications in some areas of Economics, such as Econometrics, Financial Market, Marketing and Management, and its relationships to what it has been called 'the complex paradigm'. We summarize the role played by the linear approach in Science and why it has been so successful. The use of linear models in Science has had important methodological consequences in the evolution and the concept of Science and the most important are exposed. The linear approach has been widely applied in all sciences but along the XX century, various scientific advances have questioned the omnipresence of the linear hypotheses. We briefly introduce the most important and analyze the consequences in the development of a complexity paradigm. We then center on some areas of Economics and expose the changes in models and methodologies that substantiate the validity and the applicability of this new paradigm to Economics.

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1. The Linear Hypothesis: A justification

It is not possible to analyze the evolution of the scientific knowledge without making a variety of considerations about the paper played by the linearity. In this sense, it is important to recognize that “soft” sciences (such as social sciences) were born with the purpose of adapting the pattern pointed out by the “hard” sciences (Physics fundamentally). The reason was emulating the success reached by the Newtonian thought in the explanation and prediction of natural phenomena in fields like dynamics, optics and celestial mechanics. This success was founded on a way of acting that believed in the existence of some universal, unalterable and deterministic laws that were supposed to govern the observed behavior in nature. This is the era where Laplace’s determinism, known also as “Principle of Strong Causation”, which stated that perfect predictions could be made if all the existent relationships in the nature were known, ruled.

Although conviction in determinism was strongly established in this epoch, Physics matured as a consequence of Newton and Leibniz’s works, it was common to admit, for practical purposes, the hypothesis of a lack of perfect knowledge of the existent relationships, so that the principle of strong causation was substituted by the “Principle of Weak Causation” which stated that, on average, the systems would behave as if they were deterministic and, therefore, the rules that governed their “average” behavior could be determined. Therefore, it is possible to speak about an asymptotic determinism or a tendency to the determinism more than of a real determinism. The concept of operational randomness arises in this way, since the lack of knowledge it is not an intrinsic consequence of nature, but a consequence of deficiencies in the measurement process and the presence of numerous agents in the analyzed systems. Hence, Statistics arises like a necessary tool for handling this type of chance, providing a theoretical sustenance for the taking of decisions in an uncertain atmosphere. A way to forge a rupture between the idealistic models that represented the deterministic reality, and the necessary statistical hypotheses for handling this random reality at a practical level, appears.

This double vision is in harmony with the secular tendency in western culture of linearising and simplifying, softening the vision of the real world. The roots of this tendency can be traced back to platonic philosophy that differentiated the World of Ideas (what Plato considered the real world, created by the Good and constituted by pure, symmetrical and soft forms) of the world that we inhabit a changeable, deceptive and an imperfect copy of this real world, with imperfect and rough forms, created by the Demiurgo. Hence, linear deterministic models can be associated with the Platonist World of the Ideas, and their statistical transcript with the observed world.

The development of the linear theoretical models used in the physics of XVII and XVIII centuries is based on the following principles:

- A physical phenomenon can be isolated of its environment.
- The physical laws exhibit a regular behavior.
- The interactions of the different isolated phenomena take place in an additive way or otherwise, they are considered interferences of an additive integration that do not modify very much the obtained results.
- The laboratory experiments can be repeated many times as it is necessary.

These principles stay, on average, when we work with the real world. These hypotheses justify the use of the linear models, either deterministic or statistical, for the analysis of the reality. If the interaction of the different agents does not happen in an additive way, the pattern will be considered like an interference of a linear one, since it is supposed that the results are not modified in a substantial way.

The success obtained, at a practical level, making use of these linear models in physics led to the generalization in the application of this approach in the rest of the sciences. Other more practical reasons that have also contributed to this generalization are:

- The reasonable success of the use of linear stochastic models in the practice.
- The intrinsic difficulty of non-linear models, which has supposed an authentic challenge for mathematicians until recent times.
- The impossibility of non-linear dynamic systems in the generation similar behaviors to those observed.

2. Implications of the Linear Hypothesis

The use of linear models has had various methodological consequences. The fundamental ones are the following:

1. The linear conception of reality has led to an excessively reduced vision of its dynamics, since the linear models are explosive (not controlled uncertainty) or stable. Therefore, the dynamic analysis has been traditionally centered in the stable equilibrium which it has also been generally identified with the ideal situation of the system.
2. The use of linear models leads to a simplification of the reality, only considering it in a local environment.
3. Linear models produce an excessively simple behavior so that, by themselves, they do not resemble the complex behavior of reality. It is necessary the introduction of random shocks to solve this problem. The introduction of randomness in the system is justified by the measurement problems, the lack of knowledge or the presence of excessive interacting agents. The system continues having, on average, a regular behavior.
4. Random perturbations introduced in linear models are exogenous to the system and, therefore, not informative. Hence, we talk about an exogenous complexity instead of endogenous complexity: the cause of the complexity does is not settled in model itself, but in strange factors to him. The modeled complexity is, therefore, a quantitative and not qualitative complexity, since is entirely a consequence of the presence of excessive information that, consequently, it cannot be known with accuracy by the analyst. A way to increase the complexity of the pattern is increasing the number of variables, which justifies, in turn, the introduction of random variables.
5. Because we usually work with linear models, and that the introduced interferences are not informative, the time of system does not have a prearranged direction, and it is just another variable of the system. It does not exist a time line.

3. From Linearity to Non-Linearity: The Change in the Hypotheses and Implication of the Non-Linear Hypotheses

Along the XX century, various scientific advances have questioned the omnipresence of the linear hypotheses. We can summarize them as follows:

At microscopic levels, the Heisenberg's Principle of Uncertainty questions the operational randomness and outlines the intrinsic randomness, when pointing out the existent complexity between the observed reality and the observer. At macroscopic levels, the Einstein's Theory of Relativity makes emphasis in the complexity in the relationships between the space and the time.

In the medium range, where the human being dwells, Chaos Theory shows the property of sensitive dependence to the initial conditions: there exist deterministic non-linear systems with very few variables where infinitesimal variations under the initial conditions produce completely different consequences. These systems are denominated chaotic.

This leads to very interesting consequences:

6. In these cases the Principle of Weak Causation is no longer suitable and the operational randomness substituted since by output randomness since the output is unpredictable although there is not anything random in the generating process.
7. The existence of systems with the property of sensitive dependence casts doubts on the necessity to work under a non-linear hypothesis, since non-linear systems cannot be conceived as interferences of other linear ones. Furthermore, it is no longer methodologically necessary the introduction of random shocks to generate a complex behavior, complex behavior than can be produced by chaotic systems. The system, on average, is no longer regular.
8. When working with non linear systems it makes no sense the local modelization and it is much important the study of the systems in their globality, keeping in mind the internal relationships of their elements and of these with the environment. The system is not simply the sum of the parts, and properties exhibited by the system are not exhibited by their isolated elements exist (emergent properties which leads to the concept of synergy).
9. Chaotic systems are able to generate a great variety of behaviors, and therefore the attention is no longer centered in the analysis of the stable equilibrium. The study of disequilibrium gains in importance and equilibrium is conceived like an extreme case with no great value.
10. The meaning of the complexity changes going from being a concept merely quantitative, result of the aggregation of components, to be qualitative concept intrinsic to the system. It is the own system, and not strange interferences to him, the one that generates a complex behavior.
11. The new paper of non linearity, complexity and uncertainty justifies a new vision of time also: it is an intrinsic time to the system, irreversible, a historical time, marked by the evolution of the own system which creates new relationships or destroys other existent ones.

4. Complexity and Econometrics

The linear focus led to the formulation of econometric models based on an extrinsic explanation of the real behavior of the economy. To produce such a complex behavior as the one observed it was necessary the introduction of random shocks. Despite an apparent good fit to the data, it is necessary to point out some details.

At first, the linear hypothesis has not been inherent to the underlying economic theory, but rather it has been used by reasons of analytic simplicity. Moreover, if we take into account that Economics is a social science of great complexity, it is very difficult to think about cause-effect linear relationships, on the contrary it is usually closer to reality the conviction that relationships among the economic agents are not linear, generating this way an endogenous more than exogenous complexity. The existence of chaotic non linear models has provided a great impulse to this idea since until that moment we were not aware of the great power of non linearity.

At second, taking into account a few questions related to the practical application of the linear hypothesis:

1. Despite the good fit achieved by linear models, predictions are usually bad ("paradox of prediction")
2. Residuals usually show some type of additional dependence that escapes to the linear model
3. Definitely, numerous interesting contributions of non-linear models in economic analysis derived of the quantity of different behaviors that a non linear model can capture, impossible for one linear, can be found. As Neftçi points out (Neftçi, 1986), these behaviors, usually in economics, are

- cyclical stationary behaviors; the cycle is not a consequence of interferences, but of the own dynamics of the system
- asymmetric behavior; in order to achieve the same asymmetry in linear models it is necessary to state very unrealistic hypotheses about random perturbations in the model
- volatile behaviors; with a high persistence
- "jump behaviors" incorporating thresholds
- irreversible behaviors along time (for example, as consequence of the asymmetry or the jumps)

These reasons have raised an alternative approach: the approach of non-linear disequilibrium, based on the hypothesis that the fluctuations of the system were a consequence of the dynamics of the own system, which consequently demands non-linear models of reality.

This new approach is translated into three different forms of analyzing the economic reality:

1. Development of deterministic non-linear models that produce a qualitative dynamics similar to that of the studied phenomenon, models whose purpose is not estimation (this is usually referred to as "qualitative econometrics", see Day, 1993); the analysis of bifurcations that arise in the theoretical models and the complex characterization of the system using measures of quantification of uncertainty or complexity of the series, such as Lyapunov exponents, fractal dimension and entropy, are common within this focus.

2. Development of non-linear stochastic models that, by means of their estimations, try to explain the economic reality in order to predict their future evolution. Among these models (for a revision of the topic, see Tong, 1990) those most used ones are non-linear autoregressive models (NLAR), threshold models such as linear models to pieces, polynomial to pieces, linear autoregressive (TAR), soft autoregressive (STAR), exponential autoregressive (EXPAR), and autoregressive products models (PAR).
3. Development of tools to characterize empirically time series from a complex point of view. In particular, Takens' Theorem (Takens, 1986) seeks the reconstruction of the unknown underlying dynamics that has generated the time series, so that the dynamic properties of the reconstructed system and of the unknown generating system are equivalent. This way, when computing the correlation dimension, Lyapunov exponents and entropy, in order to quantify the complexity of the reconstructed system, the complexity of the underlying system is also quantified. Lastly, various contrasts about non linearity have been developed, among those we can point out McLeod-Li (McLeod et al., 1983) and Engle (Engle, 1982) to contrast the non-linearity in variance; that of Tsay (Tsay, 1986) to contrast the non-linearity in stocking, that of Hinich (Hinich, 1982) to contrast non-linearity in third order, and the contrasts BDS (Brock et to the., 1986) and Kaplan (Kaplan, 1994 and 1995) to contrast the general hypothesis of non-linearity.

5. Complexity and Financial Markets

The modeling of financial markets and of prices of speculative products it is one of the areas of economics that it has been more developed during the XX century. The introduction in 1900 of the Brownian motion meant a revolution in the modeling of financial markets, trend that has continued during the whole XX century with the development of the required theoretical apparatus known as stochastic calculus. This has propitiated the development of the continuous-time modelling of financial markets. The Brownian motion is just the generalization in continuous time of an autoregressive model with unitary root. This stochastic process with extremely impish and subtle properties, such as the continuity of their trajectories although they are nowhere differentiable, is a complex process from a geometric point of view, complexity that is manifested in its structure fractal (Mandelbrot, 1997 and 2001). The fractal character of Brownian motion is present in the models that are based on it, models who have ruled the whole the modeling of financial markets since the second half of the XX century until the present time. Unfortunately, despite the manageability that Brownian motion has shown, what has propitiated the development of tools such as the stochastic calculus, of supreme utility and applied nowadays everywhere in the markets financial, Brownian motion represents an imprecise description of the characteristics of the financial reality. Basically five characteristics of the financial data can be pointed out that do not agree with the characteristics of the Brownian motion:

- Seemingly non-stationarity of financial data. The variations of the prices of the financial products seem to have a non stationary distribution, that is to say, periods with a lot of variations are followed by calmer periods. The Brownian motion does not portray this property, because of the identical distribution in the variation of the prices in intervals of constant duration.
- Volatility concentration. Variability is concentrated; periods where the variation of the prices is high are not isolated, but concentrated temporarily.
- Long term dependence. This characteristic of financial prices, very related the previous one, shows that variations of financial prices portrays temporal dependence that disappear

very slowly. However, Brownian motion has independent increments and therefore the variation of a period is not related with the variation of the previous period.

- Heavy tails. The distribution of the variation of financial prices are usually leptokurtic, it is appreciated empirically that big variations of prices appear more frequently than we would expect. This leads to the use of models with a greater variability of prices than the provided by the Gaussian distribution that defines the Brownian motion.

- Discontinuity. Brownian motion trajectories are continuous; there are no jump discontinuities in models where Brownian motion appears. However, this discontinuity in behavior is completely justifiable in the reality of the financial markets, very sensible to all type of information which make them extremely volatile, any variation in the economic policy, information about the evolution of the macroeconomic figures or new data about the evolution of any company makes that abrupt changes take place in the market that can be interpreted as a discontinuity.

These characteristics of financial data have made arise for new models for the financial markets.

A very important trend in the modeling of financial prices has been the introduction of the ARCH models by Engle in 1982 and Bollerslev in 1986 and its later developments. Basically, they are non-linear Gaussian models that arise to deal with the phenomena of heavy tails and variability concentration; this is done introducing variability through time in perturbations (heterocedasticity). Nelson in 1991, introduces a variation in these models, to deal with an asymmetry of financial data called “leverage” (negative surprises affect much more to the variability of financial data than positive surprises). This asymmetry in behavior is picked up in Nelson's model. Definitely, these improvement in the original ARCH model, which pursue a better fit to the real characteristics of financial series have provided us with an entire family of models such as ARCH, GARCH, EGARCH, FGARCH, FIGARCH, FIEGARCH, FIFGARCH. In Mandelbrot (1997A), and more recently in Ruiz and Pérez (2001), we can find a list of references for these models. Basically all these non-linear models make nothing but deepening in the penetration in the use of complex models in economics. The future study of these models and their relationship with the theory of complexity constitutes a very interesting and productive line of investigation.

Nevertheless these models are not exempt of problems. According to Mandelbrot (1997B) the GARCH family breaks in the consistency of the results at different time scales. The lack of scale-consistency implies that the researcher adds a restriction deciding the time-scale of the data. ARCH models do not also represent well the long-term dependence. For that purpose Mandelbrot has introduced a multifractal model based on the fractional Brownian motion and the multifractal stochastic processes. The fractional Brownian motion (FBM) is a kind of Gaussian process obtained by means of a transformation of a one-dimensional Brownian motion through a stochastic integral. This process was first considered (Shiryaev, 1999) by Kolmogorov in 1940, and Mandelbrot in 1965 and Mandelbrot and Van Ness in 1969 analyzed more thoroughly. The main features of this process are that it portrays a non-periodic recurrent variability at all the time scales and presents statistical long term dependence, so that the increments of FBM in two not overlapped intervals are always correlated, except for the case when FBM becomes a standard Brownian motion (which can be considered a particular case of FBM). It is a process that presents multiple coexistent fractal dimensions (Mandelbrot, 1997b). Multifractal processes are a generalization of self-affine processes, and present a much greater range of behaviors. This new one model in words of Mandelbrot (1997a) incorporates the regularities observed in the financial series that the model GARCH does not portray. This promising new tendency introduced by Mandelbrot deepens in the plethora of complex methods for the study of the evolution of the financial markets.

6. Complexity and Marketing

After four decades of intense development of the theory of Marketing there are still many questions that do not have a totally satisfactory answer and many the experienced difficulties in order to develop an efficient administration of marketing. On the other hand, markets are now more and more turbulent and complex, and the game rules become less clear or they change to such a speed that the adaptability and the flexibility begin to be considered by many authors the only key of the managerial success (Wilkinson and Young, 2002, Chelariu et al., 2002). The linear focus, up to now prevalent in the investigation of Marketing, it has been revealed as not very appropriate to give answer to the challenges that the new situation outlines. The corroboration of the fact that the relationship between the strategy and result are not obvious has come to show the limitations of this linear focus that it is based on the proportionality between cause and effect.

In front of this, the consideration of non-linearity stands out as a necessary change of focus. On this base new concepts have been developed such as:

1. Co-evolution: the situation of companies and products in the market can be described based on their coexistence and their continuous interactions. Individual actions of companies cannot be considered isolatedly, they establish a chain of interactions that affects their own situation and the situation of others. These interactions make possible to state (Doherty and Delener, 2001) that for each company or product it exists a share of market created by the other companies. Furthermore it is necessary to consider the interactions among all the forces or agents that take part in the market. The simultaneous co-evolution of all these elements makes that the market behavior is stationary, cyclic or even chaotic at different moments of time.
2. Auto-organization: companies are not isolated in the market but rather they form a system or business net, in which the continuous and complex interactions among their elements are those that conform the structure of the net. The positioning of each one of the companies in the market give rise to patterns of auto-organized ways of behavior operative in the net at a given instant of time (Wilkinson and Young, 2002).

The main implication for the marketing management is that the result of companies is more a consequence of their relationships and of its positioning in the net than of their own strategic actions. This way, a company also has a bounded control and a bounded ability to predict the result of its actions, which invalidates the linear principle.

Behind these two concepts there are some of the fundamental aspects provided by the new non-linear focus:

- The feedback relationships or effects in the market can be either negative or positive. The effects of negative feedback create stability and organization in the market; the effects of positive feedback create continuous change and growth (Dickson et al., 2001). The challenge for the marketing directive is to understand how these effects can happen and how the non-linear dynamics can be used for the creation of strategies.
- It is convenient to adopt a systemic perspective in front of the traditional reductionist perspective. The fundamental reason is that the system can be considered structurally divisible but functionally indivisible because it possesses properties (emergent properties) that characterize the system as a whole, but that they disappear if the system is split into pieces.

- The concept of emergent properties leads to the concept of synergy; the whole is not merely the sum of the parts, which makes us leave the idea of additivity that characterizes the linear approach.

These non-linear dynamics properties have been analyzed in several marketing systems, we can point out, for instance, works such as Herbig (1990), Fichtinger (1992), Hibbert and Wilikinson (1994) or Landa and Velasco (1997).

To conclude, we can affirm that, although the process has already begun, there is still a long way in front us in the application of non-linear approach to the Marketing Research. Diamond (1993) has studied the influence that non-linear dynamics may have on theoretical aspects of Marketing Research. In our opinion, the aspects in which the investigation in Marketing in connection which non-linearity will be centred in next years are the following ones:

1. It is necessary to revise the models that constitute the base to the management of the relationships with the market, to give entrance to some non-linear models that can consider a wide variety of complex behaviors. The objective of the introduction of these non-linear models is to permit to the responsible for marketing to have a wider range of models that allows him to give a better explanation to the unexpected behaviors that often happen in the consumption markets.
2. It is not generally possible to predict the future evolution of the market, because small changes under the initial conditions can produce very different consequences. However, the construction of non-linear models can help to understand the consumer's behavior from a qualitative point of view. This focus can result more interesting than the quantitative one, given the changing evolution of the markets. (Diamond, 1993).
3. An analysis of the administration models from the perspective of the non-linear dynamics can help to anticipate and/or to avoid chaos. The chaotic systems can, sometimes, be controlled. Especially in those cases where the parameters related to the stability of the system and the occurrence of the chaos are under the control of the company (Herbig, 1990, Hibbert and Wilkinson, 1994, Landa and Velasco, 1997). One of the objectives of this analysis would be the discovery of the range of values of the parameters that the decision-maker should avoid in order to minimize the appearance of chaotic dynamic, or to help him to establish the limits of the stability of the system.

7. Complexity and Management

The evolution in the scientific thought and in the own managerial world shows the necessity of a new approach focus that lead us to think in terms of non-linear systems. The technocratic management models must leave room, therefore, to a strategic management where the values of the new economics of complexity are assumed (Nieto de Alba, 2000).

While traditional management deals with the idea of centralized, hierarchized and nested organizations, where information flows from the top to the bottom, in the complex management, organizations are considered less bureaucratic and centralized, the information flows up and down in any direction. This has necessarily changed the concept of company as a consequence of the new perception of internal and external relationships in the company, to the dynamism of the contemporary society and the globalization of markets: the environment changes at a great speed, and it is not only necessary a quick adaptation to this change, but a different kind of adaptation.

Under a traditional perspective, management deals with a stable environment, with the following characteristic (Nieto de Alba, 1999):

1. The goal of management is to preserve equilibrium between the organization and its environment, facing closed and predictable changes where what will happen is previously known.
2. Management is reactive; the adaptation to the change is a reaction activated from the past.
3. It is an unenthusiastic management, with great aversion to the risk and dependence of the formal control
4. It is a bureaucratized and tightly controlled management, what generates restrictive limits among the organizational levels.

On the contrary, management based on the complex and non-linear paradigm has the following significant aspects (Nieto de Alba, 1996):

1. It is a creative and innovative management: the creation of a new order that emanates from disorder is considered and the future of the organization is created taking into account this principle. The goal of this kind of management is to create new behavior rules and to open new markets, writing their own rules in an environment of uncertainty without answers.
2. The importance of confusion and the flexible behavior, given the turbulent environment in which the company is moving, is recognized. The instability is essential in the business management. We are dealing with a bounded instability where the success of organizations considered as systems of non-linear feedback and “no-equilibrium” dwells.
3. The management is based on the complex learning in group, with no bureaucracies and no established limits, but spontaneous and self-structured learning, based on the cooperation and in the dialogue with the environment (Kiel, 1994). This learning is summed up in the pursuit of strategies and in the conformation of the culture of the organization, the planning, the control and learning are simultaneous since uncertainty shortens the periods of decision.
4. It is a strategic management, where the importance of disorder, conflict and instability as sources of creative strategies is highlighted. It is necessary to pay attention to new concepts, born in the non linearity culture: endogenous variables, global systems, feedback phenomena, amplification and possible turbulences that can be generated as a consequence of the taken decisions.
5. The management is based on the search of general qualitative models, so that we can deal with similar situations to those experienced, since the changing environment makes no possible to know the long term future of the company, even in statistical terms (Stacey, 1994), and consequently tight planning makes no sense (Stilwell, 1996). Instead of planning new favorable conditions should be created (Stacey, Griffin and Shaw, 2000)
6. Management must be flexible, avoiding autoritarism and formal groups; at the same time the polyvalence of employees, the self-managed groups and the existence of opposed forces looking for the generation of information and instability is encouraged (Navarro Cid, 2000)

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