Generic Structures:
First-Order Positive Feedback

Produced for the
System Dynamics in Education Project
MIT System Dynamics Group

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1. Introduction

Generic structures are relatively simple structures that recur in many diverse situations. In this paper, for example, the models of a bank account and a deer population are shown to share the same basic structure! Transferability of structure between systems gives the study of generic structures its importance in system dynamics.

Road Maps contains a series of papers on generic structures. In these papers, we will study generic structures to develop our understanding of the relationship between the structure and behavior of a system. Such an understanding should help us refine our intuition about the systems that surround us and allow us to improve our ability to model the behaviors of systems.

We can transfer knowledge about a generic structure in one system to understand the behavior of other systems that contain the same structure. Our knowledge of generic structures and the behaviors they produce is transferable to systems we have never studied before!

It is often the case that the behavior of a system is more obvious than its underlying structure. Systems are then referred to by the common behaviors they produce. However, it is incorrect to assume such systems are capable of exhibiting only their most popular behaviors, and we need to look more closely at the other behaviors possible. In effect, our study of generic structures examines the range of behaviors possible from particular structures. In each case, we seek to understand what in the structure is responsible for the behavior produced.

This paper introduces a simple generic structure of first-order linear positive feedback. We illustrate our study of the positive feedback structure with many examples of systems containing the structure. You will soon begin to recognize the structure in many of the models you see and build. In the exercises at the end of the paper, we provide you with an opportunity to see how you can transfer your knowledge between different systems.

2. Exponential Growth

Exponential growth is produced by a positive feedback loop between the components of a system. The characteristic behavior of exponential or compound growth is shown in Figure 1. Many systems in the world exhibit the exponential behavior of a process feeding upon itself. For example, in an ecological system, the birth of deer increases the deer population, which further increases the number of deer that are born. At your bank, your account balance is increased by the interest you earn on it, and the larger your balance gets, the more interest you earn on it! Another system which can be said to exhibit exponential growth is the knowledge-learning system. Simply stated, the more you know, the faster you learn, and then gain even more knowledge.

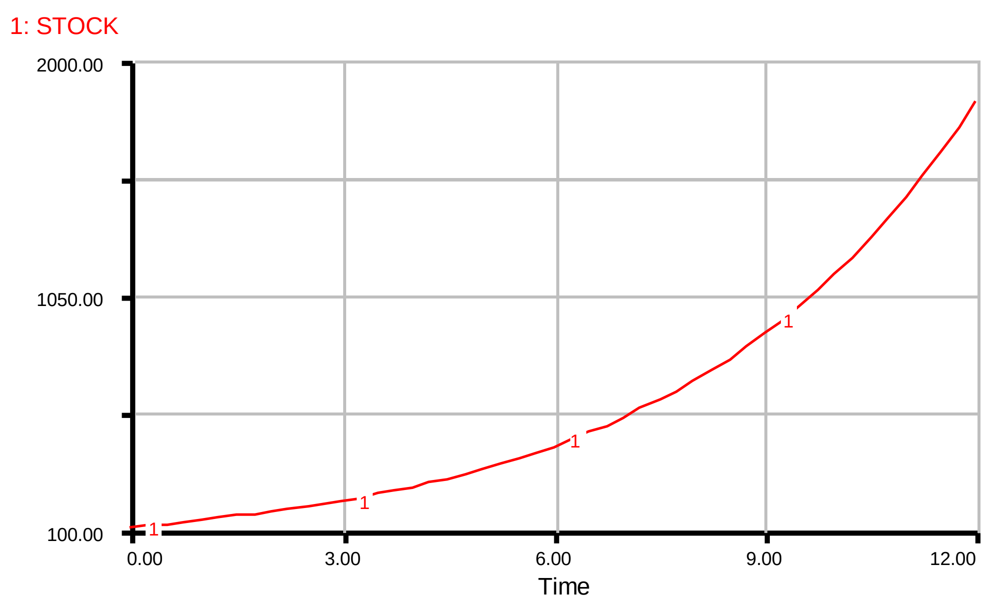

Figure 1 Exponential Growth Curve

These very different systems exhibit the same behavior pattern because the relationship between their components is fundamentally the same. They all contain the first-order linear positive feedback generic structure. Population is related to births in the same way your bank balance is related to the interest it earns and knowledge is related to learning.

Let us begin to explore the nature of this relationship by looking at the structure of our three example systems.

2.1. Example 1: Population-Birth system

Our first example shown in figure 2 is taken from the ecology of a deer population. The **deer population** is the stock, and the **births** of deer is the net inflow to the stock. The amount of deer births is equal to the amount of female deer that reproduce and is calculated as a compounding fraction (called **birth fraction**) of the total deer population. The

births = deer population * birth fraction.

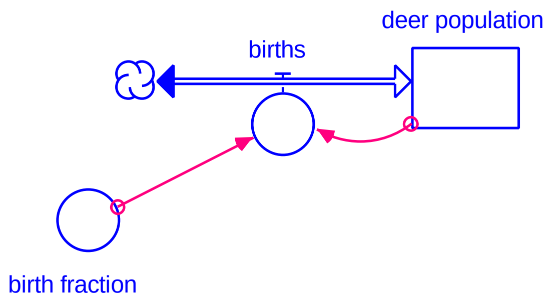

Figure 2 Model of a population-birth system

2.2. Example 2: Bank balance-interest system

Our second example in figure 3 shows the relationship between a bank balance and the interest it earns. The **bank balance** is the stock, and the **interest earned** is the inflow to the stock. The amount of interest earned every year is equal to a compounding fraction (**interest rate**) of the bank balance. The

interest earned = bank balance * interest rate.

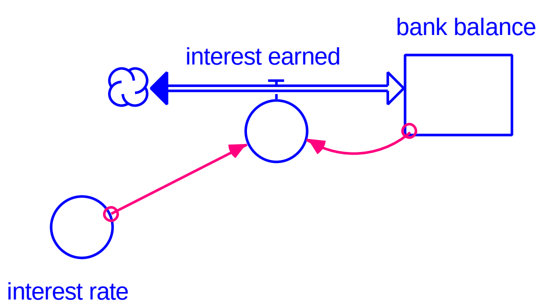

Figure 3 Model of a bank balance system

2.3. Example 3: Knowledge-learning system

This third example is of a more abstract system. Figure 4 shows that the stock of **knowledge** is increased by the net inflow **learning**. The rate of learning is the knowledge spread out over the **time to learn**. Therefore, learning is equal to the knowledge divided by the time to learn. The time to learn is known as the time constant of the system. Basically, the more you know, the faster you learn. The

learning = knowledge/time to learn.

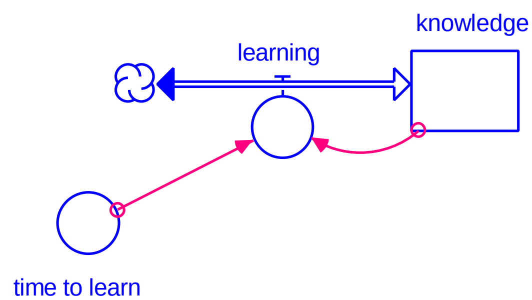

Figure 4 Model of a knowledge-learning system

Note that in the equation of the rate, we divide the stock (knowledge) by time to learn. This is analogous to multiplying by a fraction as seen in examples 1 and 2. The units for time to learn are the units of time like weeks or months. The units of the compounding fraction would be unit/unit/time.

As is evident from Figures 2, 3, and 4, all three of these systems have essentially the same structure.

3. The Generic Structure

We will now study the generic structure and then explore the possible behavior it can produce.

3.1. Model Diagram

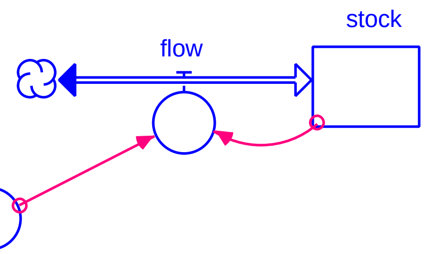

Figure 5 Model of the underlying generic structure

The model diagram of the first-order positive generic structure is shown in figure 5. In the equation of the rate, we multiply the stock by the compounding fraction or divide the stock by the time constant. The time constant is simply the reciprocal of the compounding fraction.

3.2. Model Equations

The equations for the generic structure are

stock(t) = stock(t - dt) + (flow) * dt

DOCUMENT: This is the stock of the system. It corresponds to the deer population, the bank balance, and the stock of knowledge in the examples above.

UNIT: units

INFLOWS:

flow = stock*compounding_fraction

DOCUMENT: The flow is the fraction of the stock that flows into the system per unit time. It corresponds to the births, the interest earned, and the learning in the examples above.

UNIT: units/time

compounding_fraction = *a constant*

DOCUMENT: This is the compounding fraction or growth factor. It determines the inflow to the stock. The compounding fraction corresponds to the birth fraction and the interest rate in the examples above. It is the amount of units added to the stock for every unit already in the stock, every time.

UNIT: units/unit/time

Note: If we had a time constant instead of a compounding fraction the equation for the flow and the time constant would be

INFLOWS:

flow = stock/time constant

UNIT: units/time

time_constant = *a constant*

DOCUMENT: This is the time constant. It is the adjustment time for the stock. It corresponds to the time to learn in the above example. This is the time for each initial unit to compound into a new unit.

UNIT: time

From the comparison of the two possible equations for the rate, we notice that the multiplier in the rate equation is given by

$$\text{multiplier (for the stock) in the rate equation} = \text{compounding fraction} = \frac{1}{\text{time constant}}$$

3.3. Model Behavior

The characteristic feature of exponential growth is its constant doubling time, i.e. the time it takes for the stock to double remains constant. For example in Figure 6, it takes the stock 7 years to double from 100 to 200 and also the same time to double from 800 to 1600!

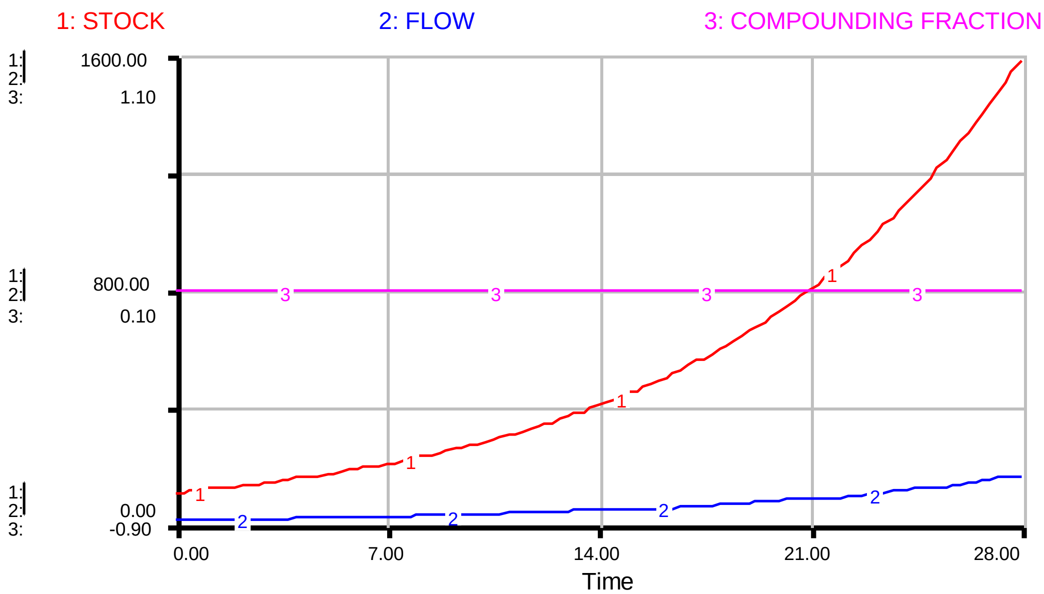

Figure 6 Results of a simulation of the positive feedback generic structure.

To find the doubling time of the stock, we need the time constant of the system. The time constant may either be given to you directly (as the time to learn in Example 3 above), or if you have a compounding fraction, the time constant is simply it's reciprocal. The time constant is obtained from the compounding fraction by

$$\text{Time constant} = \frac{1}{\text{compounding fraction}}$$

The doubling time for the stock is given by

Doubling time ? 0.7 ? Time constant [1]

4. Behaviors produced by the generic structure

To explore the different behaviors possible, let us first experiment by changing the initial value of the stock and keep the value of the compounding fraction constant. The different behaviors for changing values of the initial stock are shown below. The stock is given initial values of -200, -100, 0, 100, and 200 for runs 1 through 5 respectively. The compounding fraction is kept constant at 0.1. The results are shown in Figure 7.

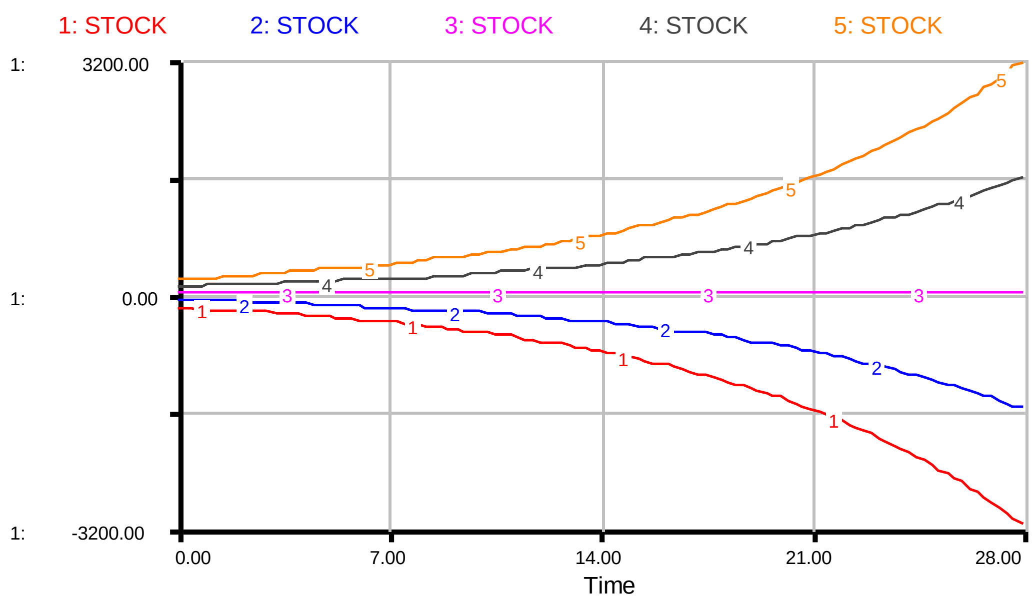

Figure 7 Simulation for different initial values of the stock

The flow is a constant fraction (compounding fraction) of the stock. As the stock increases, the compounding fraction remains the same, but now, the flow is the fraction of a larger stock, and therefore the flow increases with the stock. The slope of the stock at a

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[1] ln 2 is approximatly equal to 0.7.

point in time is equal to the net flow into it at that time. Therefore, for each curve, the slope of the stock is increasing or decreasing as the stock is increasing or decreasing.

With a positive value for the compounding fraction, the nature of the behavior is determined by whether the initial value of the stock is positive or negative. For the loop to be a positive feedback loop, we require that the compounding fraction be positive[2].

Therefore, we see that the generic structure of a first-order positive loop can exhibit three types of behavior - positive exponential growth, unstable[3] equilibrium, and negative exponential growth.

Let us now explore what accelerates or retards the exponential growth of a system. We will study the effect of changing the value of the compounding fraction while keeping the initial value of the stock constant. The compounding fraction is given values of 0, 0.1, 0.2, 0.3, and 0.4 for runs 1 through 5 respectively. The initial value of the stock is kept constant at 100. The change in rate of exponential growth is shown in Figure 8.

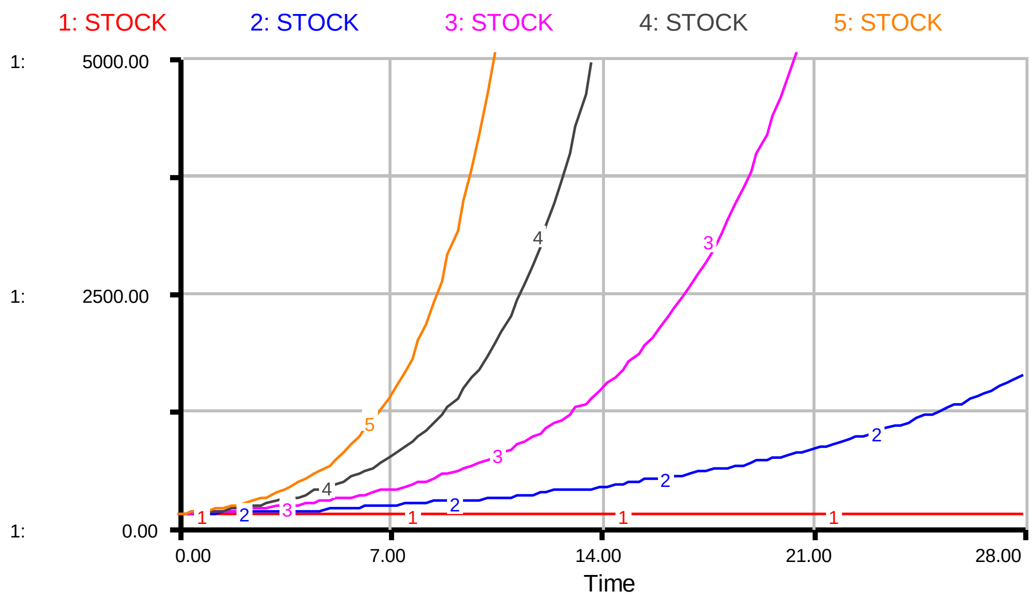

Figure 8 Simulation for different values of the compounding fraction

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[2] A positive compounding fraction is required for a positive feedback (reinforcing) loop as it gives the net flow the sign of the stock (either positive or negative). A negative value for the compounding fraction will make the loop a negative feedback or balancing loop.

[3] This equilibrium is called unstable as the slightest deviation of the value of the stock away from zero will destroy the equilibrium and result in exponential growth.

The slope of the stock at a specific point[4] in time is equal to the net flow into it at that time. The flow is a larger fraction of the stock for a larger compounding fraction. Therefore, the slope of the stock is greater for a larger compounding fraction.

The larger the compounding fraction is, the larger the flow and the faster the growth of the stock. A larger compounding fraction accelerates exponential growth.

For a negative initial value of the stock, the effect of the compounding fraction on growth rate is the same, except the growth is in the negative direction.

5. Summary of important characteristics

Structure:

The loop is a positive feedback loop if and only if the stock has a positive sign in the equation for net flow into the stock. A positive sign in the equation for flow gives the flow the same sign as the stock (reinforcing behavior). Therefore, the simplest positive feedback loop requires a positive compounding fraction for the inflow to the stock.

Behavior:

We summarize the behavior of the positive feedback loop in table 1 below. Although positive feedback loops are best known for their exponential growth, they do exhibit other behaviors. Remember: A negative multiplier in the rate does not create a positive feedback loop.

The generic structure of a first-order positive loop can exhibit three types of behavior - positive exponential growth, unstable equilibrium, and negative exponential growth.

For an initial value of the stock and the multiplier in the rate (fraction or time constant), the behavior of the stock is given in italics	**Stock**		
	Negative	**Zero**	**Positive**

[4] The slope of the stock at a point is the slope of the line tangent to the curve at that point.

Compounding Fraction[5]	**Zero**	*Equilibrium*	*Zero*	*Equilibrium*
	Positive	*Negative Exponential growth*	*Zero*	*Positive Exponential growth*

Table 1 Summary of the behavior of a positive feedback loop

Exponential growth requires an initial value of the stock other than zero. Exponential growth has a constant doubling time. The rate at which an exponential growth occurs increases with the value of the compounding fraction.

Look over table 1 and the graphs of the simulation runs till you internalize your knowledge about positive feedback loops. When you feel confident about your understanding of the behavior, you should go on to the exercises in the next section.

6. Using insights gained from the generic structure

We have seen examples of different systems with the same positive feedback loop structure. We studied the underlying generic structure to develop our intuition about positive loops. Now, we apply the insight we gained from the generic structure to understand the behavior of other systems.

To do the exercises below, you need not simulate the models; hand computation should suffice. However, after answering the questions we encourage you to build and experiment with the models.

[5] Zero compounding fraction corresponds to an infinite time constant. This is not a situation you will be confronted with.

6.1. Exercise 1: Software sales

The customer base of a software manufacturer increases with the addition of new customers. Through the word of mouth, a fraction of the present customers encourage other people to become new customers. The model for this simple positive feedback system is shown below.

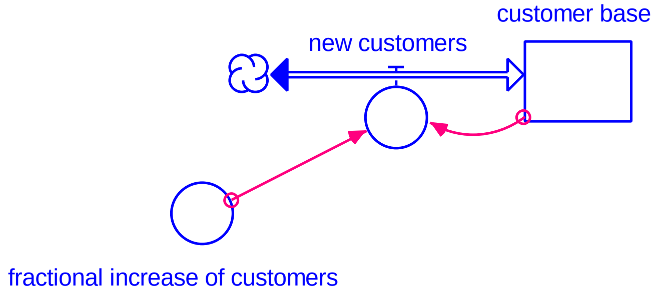

Figure 10 Model for Software sales

There are two software companies, Nanosoft and Picosoft, each of which have a customer base consisting of 10,000 customers, and a fractional increase is 0.1 customers/customer/week (the fraction means that 1 out of 10 customers convinces another person to become a customer each week).

1. What is the time constant and doubling time? Give their units. What are the units of new customers?
2. Approximately how much time does it take for the customer base of Nanosoft to grow to 40,000 customers?
3. If Nanosoft wants to have 80,000 customers in the same amount of time, how could it change the initial value of the stock to achieve this?[6]
4. Picosoft also wants 80,000 customers in the same time but decides to change the fractional increase to achieve this. What change should it make?
5. If Nanosoft has a customer base three times larger than Picosoft, which of the two firms do you think will grow faster? What is the ratio of their customer bases after 14 weeks?

[6] Although changing the initial stock may not be a feasible option in the real system, our purpose here is to understand the effect of different initial values of the stock on its growth.

6.2. Exercise 2: Making Friends

Brenda and Brandon are twins who have just moved into a new town to live with their aunt. Although they are twins, their personalities are quite different. Brenda is very sociable and makes friends easily. She usually makes a new friend, though each friend she already has, every 2 weeks. Brandon, on the other hand, is quite shy; it usually takes him twice as long as Brenda to make a new friend through each of his current friends.

In this new town, Brandon already has 5 friends that he made in previous summer visits. Brenda, however, has never been to this town before and the only 'friend' she has here is her aunt.

Figure 11 is a very simple model of the process by which new friends are made. The model indicates that the rate at which a person makes new friends depends on the amount of friends that a person already has and the time to make a new friend. For example, if Brenda has a lot of friends, she will be introduced to a lot of new people (friends of friends), and if she doesn't take much time to make friends with a new person, then she will make a lot of new friends very quickly.

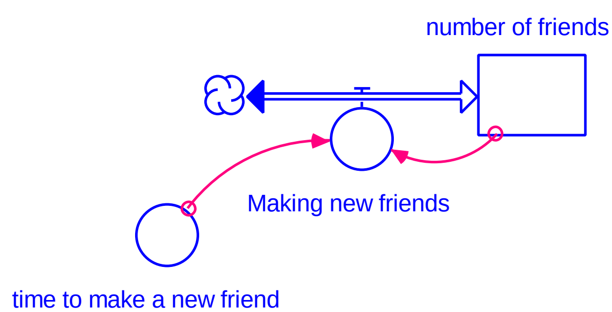

Figure 11 Model for making friends

1. What is the time constant and doubling time for Brenda?
2. What is the time constant and doubling time for Brandon?
3. By the time school starts (9 weeks after moving in) who will have more friends, Brenda or Brandon? You don't need to find exactly how many friends Brenda and Brandon have after 9 weeks; just provide an indication of who has more friends after 9 weeks.

6.3. Exercise 3: Account balance

Brandon decides he has had enough of school, and plans to start his own software business to compete with Nanosoft and Picosoft. We have built a model of his account balance shown in figure 12. A loan is considered as a negative account balance.

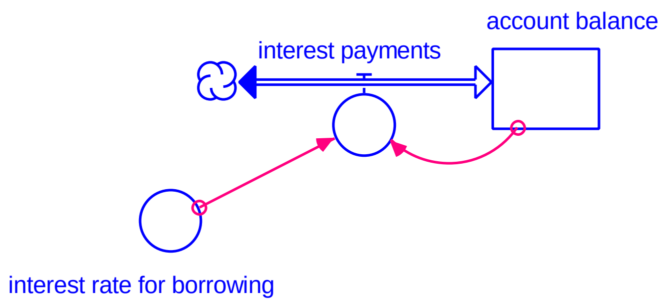

Figure 12 Model for account balance

To buy a computer, he takes out a $2,000 loan from his bank at an interest rate of 5%. On his way home from the bank, he meets Brenda who tells him of another bank that will forgive $1,000 of his loan if he transfers to them. This bank charges an interest rate of 10%. Brandon doesn't understand exponential growth very well and is confused about what he should do. What do you recommend? What will his account balance be after 14 years for each bank?

7. Solutions to Exercises

7.1. Answer to Section 6.1: Software Sales

1. The time constant $= \dfrac{1}{\text{fractional increase}}$, or 10 weeks.

The doubling time = 0.7 x the time constant, or 7 weeks.

2. For the customer base of 10,000 to grow to 40,000, the initial base doubles twice. Each doubling time is 7 weeks, thus it takes 14 weeks for the firm to reach 40,000 customers.

3. Since the time to reach 80,000 is given as 14 weeks (two doubling times), the initial value for the stock should be 20,000 customers.

4. To reach 80,000 customers, a base of 10,000 doubles <u>three</u> times in the 14 weeks. Working backwards:

1 doubling time = $\frac{14}{3}$ or $4.\bar{6}$ weeks.

1 time constant = $\frac{\text{doubling time}}{0.7}$ = $6.\bar{6}$ weeks.

Fractional increase = $\frac{1}{\text{time constant}}$ = $\frac{21}{140}$ or .15 customers/cust/week

5. Nanosoft grows faster since it has a larger stock than Picosoft. While the fractional increase of each company is the same, the rate of growth for Nanosoft is larger because the actual number of customers the fractional increase corresponds to is larger. The ratio of customer bases after 14 weeks is still the same, 3 to 1. It may seem impossible for the ratio to remain the same while one firm grows at a faster rate than the other. The key to understand is that Nanosoft does grow faster but also has a larger distance to go to maintain the ratio of 3 to 1.

7.2. Answer to Section 6.2: Making Friends

1. Brenda's time constant is her time to make a new friend, which equals 2 weeks. The doubling time is the time constant x 0.7, or 1.4 weeks.

2. Brandon takes twice as long to make friends. His time to make a new friend is twice that of Brenda's. His time constant is thus 4 weeks. The doubling time is the time constant x 0.7, or 2.8 weeks.

3. The easiest way to answer this question is to use the doubling times and the initial values for the stocks of friends and make a small chart of Brenda and Brandon's number of friends for the summer.

Week	Brenda's Friends	Brandon's Friends
0	1	5
1.4	2	—
2.8	4	10
4.2	8	—
5.6	16	20
7.0	32	—
8.4	64	40

9	more than Brandon	less than Brenda

Because of the nature of the positive generic structure, once Brenda has more friends that Brandon, she will always have more friends that Brandon. We can then infer that in week 9, the end of the summer, Brenda has more friends than Brandon.

7.3. Answer to Section 6.3: Account Balance

Brandon's bank's interest rate is 0.05. The time constant is equal to 20, and the doubling time of the debt is equal to 14 years. The other bank's interest rate is 0.1 and has a time constant of 10 years. The doubling time of a debt is 7 years.

Years	Debt in Brandon's Bank	Debt in Other Bank
0	2000	1000
14	4000	4000
28	8000	16000
42	16000	64000
56	32000	256000

This chart clearly illustrates the power of exponential growth. The bank which Brandon should invest in depends on when he plans on paying back his loan. If he plans to pay back in the first 14 years, then the other bank would save him money. If it will take Brandon over 14 years to repay the loan, the bank he already borrowed from is his best bet.

8. Appendix - Model Documentation

8.1. Documentation for section 2.1: Population-birth system

deer population(t) = deer population(t - dt) + (births) * dt

INIT deer population = 100

DOCUMENT: This is the number of deer present in the system.

UNIT: deer

INFLOWS:

births = deer population * birth fraction

DOCUMENT:This is the number of deer born every year.

UNIT: deer/year

birth fraction = .3

DOCUMENT: This is the number of deer born per deer every year.

UNIT: deer/deer/year

8.2. Documentation for section 2.2: Bank balance-interest system

bank balance(t) = bank balance(t - dt) + (interest earned) * dt

INIT bank balance = 100

DOCUMENT: This is the amount of money in a bank account

UNIT: dollars

INFLOWS:

interest earned = bank balance * interest rate

DOCUMENT: This is the amount of interest earned per year on the money in the account.

UNIT: dollars/year

interest rate = .025

DOCUMENT: This is the number of dollars earned per dollar in 1 year.

UNIT: dollars/dollar/year

8.3. Documentation for section 2.3: Knowledge-learning system

knowledge(t) = knowledge(t - dt) + (learning) * dt

INIT knowledge = 100

DOCUMENT: This is the amount a person knows, measured in facts about a subject.

UNIT: facts

INFLOWS:

learning = knowledge /time to learn

DOCUMENT: This is the rate at which new facts are learned per day.

UNIT: facts/day

time to learn = 3

DOCUMENT: This is the time constant of the system. It takes an average of 3 days for each fact to assist in the learning of a new fact.

UNIT: day

Vensim Examples:
Generic Structures: First-Order Linear Positive Feedback

By Aaron Diamond

October 2001

2.1 Example 1: Population-Birth system

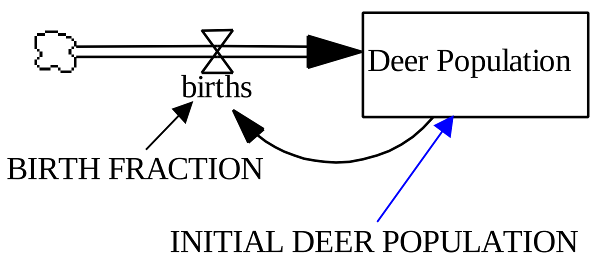

Figure 13: Vensim Equivalent of Figure 2: Model of a population-birth system

Documentation for population-birth model

(1) BIRTH FRACTION=0.3
Units: deer/deer/year
This is the number of deer born per deer every year.

(2) births=Deer Population*BIRTH FRACTION
Units: deer/year
The flow is the number of deer born every year.

(3) Deer Population= INTEG (births, INITIAL DEER POPULATION)
Units: deer
This is the number of deer present in the system.

(4) INITIAL DEER POPULATION=100
Units: deer

(5) FINAL TIME = 28

Units: year

The final time for the simulation.

(6) INITIAL TIME = 0

Units: year

The initial time for the simulation.

(7) SAVEPER = TIME STEP

Units: year

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: year

The time step for the simulation.

2.2 Example 2: Bank balance-interest system

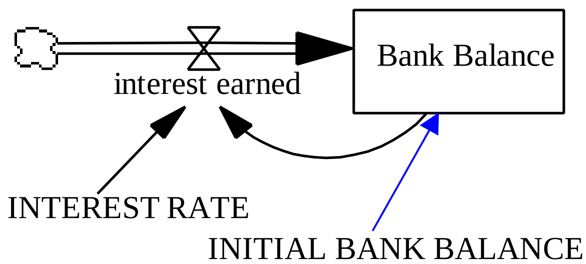

Figure 14: Vensim Equivalent of Figure 3: Model of a bank balance system

Documentation for bank balance model

(1) Bank Balance= INTEG (interest earned, INITIAL BANK BALANCE)
Units: dollars
This is the amount of money in a bank account.

(2) FINAL TIME = 28
Units: year
The final time for the simulation.

(3) INITIAL BANK BALANCE=100
Units: dollars

(4) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(5) interest earned=Bank Balance*INTEREST RATE
Units: dollars/year
The flow is the amount of interest earned per year on the money in the account.

(6) INTEREST RATE=0.025
Units: dollars/dollars/year
This is the number of dollars earned per dollar in 1 year.

(7) SAVEPER =TIME STEP
Units: year
The frequency with which output is stored.

(8) TIME STEP = 0.0625
Units: year
The time step for the simulation.

2.3 Example 3: Knowledge-learning system

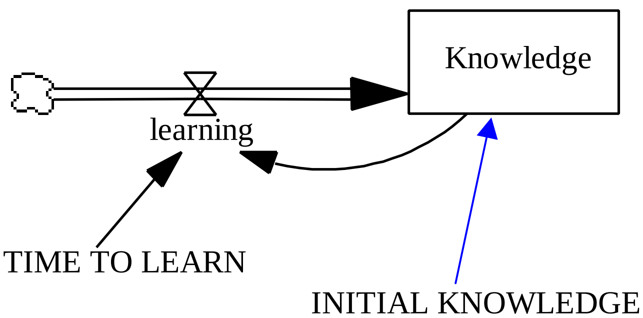

Figure 15: Vensim Equivalent of Figure 4: Model of a knowledge-learning system

Documentation for knowledge-learning model

(1) FINAL TIME = 28
Units: day
The final time for the simulation.

(2) INITIAL KNOWLEDGE=100
Units: facts

(3) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(4) Knowledge= INTEG (learning, INITIAL KNOWLEDGE)

Units: facts

This is the amount a person knows, measured in facts about a subject.

(5) learning=Knowledge/TIME TO LEARN

Units: facts/day

This is the rate at which new facts are learned per day.

(6) SAVEPER =TIME STEP

Units: day

The frequency with which output is stored.

(7) TIME STEP = 0.0625

Units: day

The time step for the simulation.

(8) TIME TO LEARN=3

Units: day

This is the time constant of the system. It takes an average of 3 days for each fact to assist in the learning of a new fact.

3.1. Model Diagram

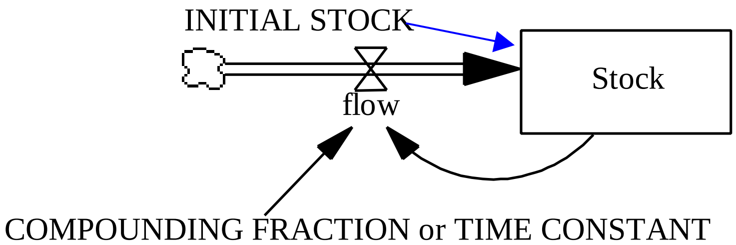

Figure 16: Vensim Equivalent of Figure 5: Model of the underlying generic structure

Documentation for generic structure model

(1) COMPOUNDING FRACTION or TIME CONSTANT=a constant
Units: units/units/time for COMPOUNDING FRACTION,
Units: time for TIME CONSTANT
This is the compounding fraction or growth factor. It determines the inflow to the stock. The compounding fraction corresponds to the birth fraction and the interest rate in the examples above. It is the amount of units added to the stock for every unit already in the stock, every time.

(2) FINAL TIME =28
Units: Month
The final time for the simulation.

(3) flow=Stock*COMPOUNDING FRACTION, or Stock/TIME CONSTANT
Units: units/time
The flow is the fraction of the stock that flows into the system per unit time. It corresponds to the births, the interest earned, and the learning in the examples above.

(4) INITIAL STOCK=a constant
Units: units

(5) INITIAL TIME = 0

Units: Month

The initial time for the simulation.

(6) SAVEPER =TIME STEP

Units: Month

The frequency with which output is stored.

(7) Stock= INTEG (flow, INITIAL STOCK)

Units: units

This is the stock of the system. It corresponds to the deer population, the bank balance, and the stock of knowledge in the examples above.

(8) TIME STEP = .0625

Units: Month

The time step for the simulation.

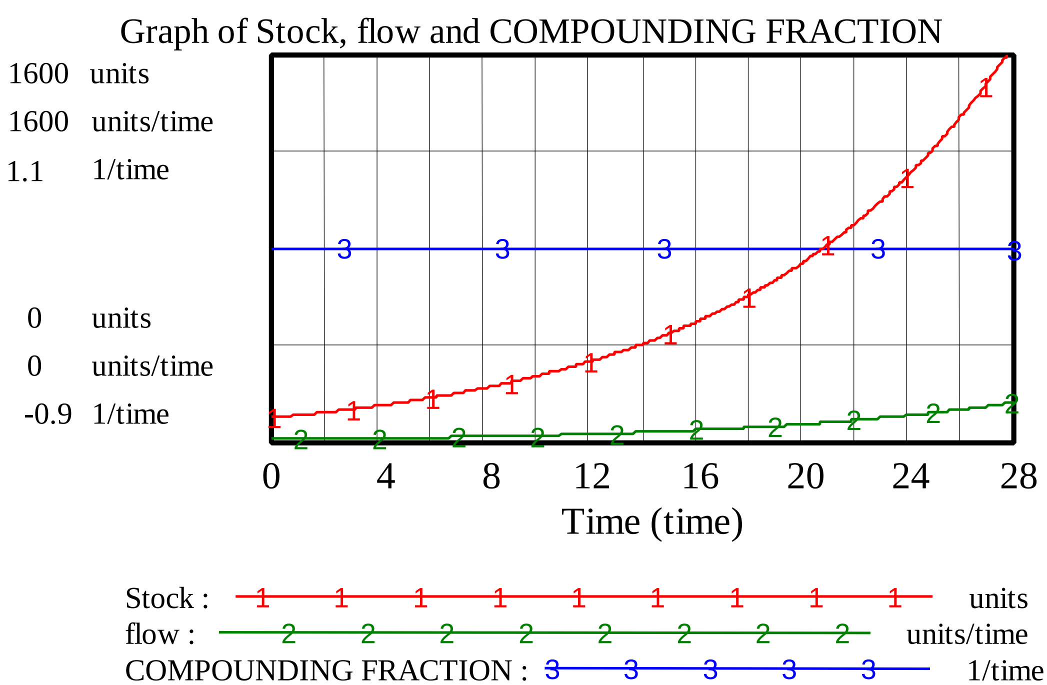

Figure 17: Vensim equivalent of Figure 6: Results of a simulation of the positive feedback generic structure.

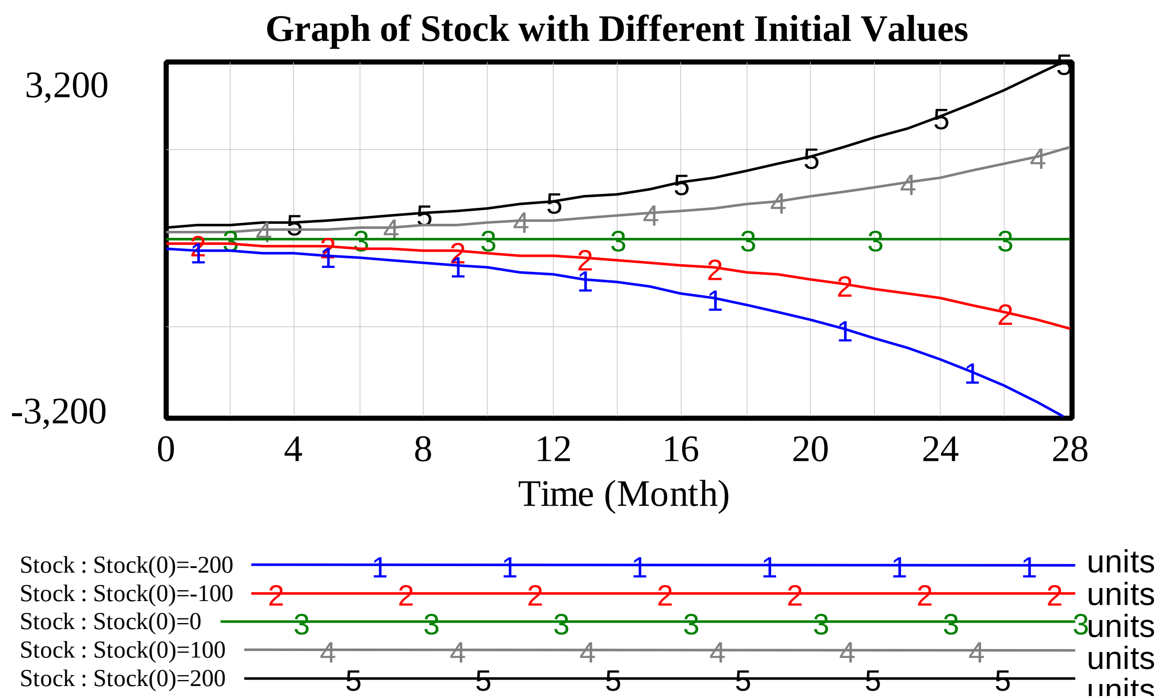

Figure 18: Vensim Equivalent of Figure 7: Simulation for different initial values of the Stock

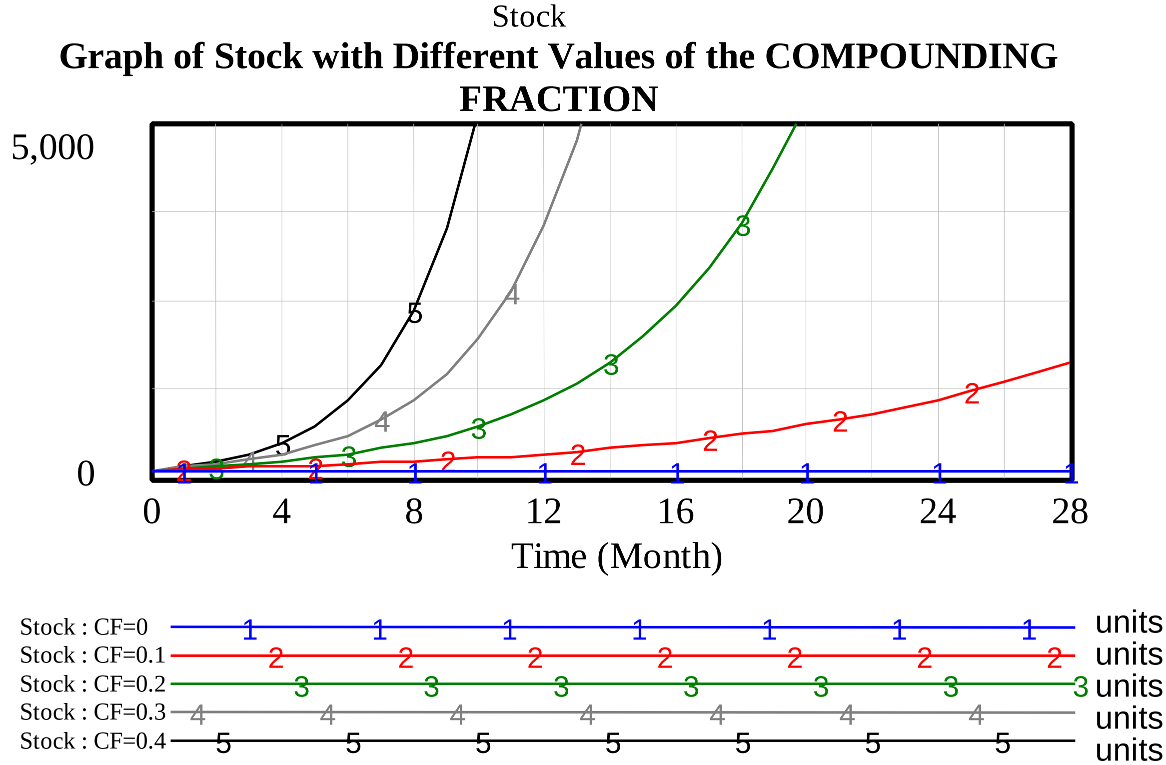

Figure 19: Vensim Equivalent of Figure 8: Simulation for different values of the COMPOUNDING FRACTION