# Mathematical Representation of Social Systems. Uncertainty and Optimization of Social System Evolution

I. A. Miklashevich

*Department of Theoretical Mechanics, Belarusian National Technical University, 65 Skaryna Ave., Minsk, 220027, BELARUS*

E-mail: miklashevich@yahoo.com

(Received 13 August 2002)

The development of social systems as an evolution of systems with inside non-determination is studied on the basis of non-standard analysis. The evolution of social systems is represented as a movement in the space of states. "Human" states space is the reduction of full space which is happening by the filtration. The global and local optimal trajectories from the point of view of mathematics are defined.

**Key words:** control, hierarchy, social system, hazy, optimal evolution, trajectory of development

**PACS numbers:** 89.20; 89.65.EF; 89.75.FB; **MSC:** 91 B66; 91 B74; 91 B 99

## 1 Introduction: Mathematics and social system modeling - necessity or excess?

The problem of sustainable development of the civilization becomes more and more important. It is a pity that the humanity looses the global fight for water, ground, food. In this failure there is the origin of future conflicts, which can be a source of future local wars and dictator regimes with a civilized surrounding hidden.

That is why the forecasting of the development of large social systems in a long perspective (ten and more years) from the abstract-theoretical problem becomes a practical one. The main sphere of using such models is the evolution control of social systems with the aim of optimization of the trajectory, definition of the development process control parameters and minimizing the necessary action in the vicinity of evolution critical points. It is well known that social system evolution is not controlled by the second law of thermodynamics and the system development does not lead to simplification but to complication of system structures. This complication demands introduction negentropy rather than entropy as a main parameter of system evolution. In any case we can study the problem of social evolution as a specific case of optimization problem with the specific rule of interaction. Taking into account social system description the most important problem consists in determination of minimal influence (minimal act) of system coordinator to achievements of coordinator tasks. This minimal influence is most important near the critical point of evolution, where the system selects one of possible ways of prospective evolution. From mathematical point of view, this critical point corresponds to bifurcation point. The critical point may have different nature depending on the set of principal variables in the space of states. Classical "social critical point" is the selection of one of different possibilities for economical model of system development. This selection has also economical effect: it is easier to prevent crisis rather than to overcome it. Group of local conflicts of last time is an another example of social evolution near the critical point with a very complicated set of outcomes: political, economi-

cal, humanitarian.

This reality supports us in the development of general mathematical description of social processes and social system evolution. Some aspects of this problem is well known for physicists and the mathematicians [6, 13, 14]. As a rule the description of social and living systems is realized on the base set of phenomenological equations which are linked with the activity of systems. This description gives an opportunity to research only some aspects of full activity of living systems. Proposed description based on statistically - informational approximation of living system [2, 9] and general system theory [17, 18], taking into account the Aed realization [12] for non-standard systems.

## 2 Social systems with inside non-determination

Understanding the problem of social systems modeling is connected with some methodological difficulties. One of these difficulties is the creation of necessary mathematical apparatus for describing systems with inside uncertainty. We define the system with inside uncertainty as a system which has in its description uncertainties caused by some reasons rather than uncertainties due to methods of description. In the simplest case we can consider uncertainty of a system as changing the results of the activity of the system in various range of changing.

As a rule most of social systems are systems with uncertainty.

**Definition 1**. *In the wide meaning under the phrase "social system" we imply objects which are connected with collective actions of living organism, objects, which do not exist out of borders of activity of the collective.*

Our goal is the investigation of more narrow social systems and systems connected with activity of human society.

Social systems which are connected with acts of human form only the part of possible social systems, but the complicated one. The main difficulties of forecasting and creation the adequate models of social systems are connected with the acts of people and are raised by the next conditions:

**A.** This social systems are systems with uncertainty because units of social systems (people) have mind and freedom of will.

**B.** The origin of new properties of the system are not connected with the previous system evolution history (the future state does not follow from previous one).

**C.** Systems are non-determined and onset of chaos regimes or regimes of self-organization in them is possible. The transition between chaos and self-organization is possible.

Our interpretation of social systems is rather simple but it helps to widen all diversity of activity manifestation of a living organism - from self-organization of structures in communities of microorganisms to chaos processes in economics. From the mathematical point of view each system which satisfies the determination of social system generates the specific generalized space of states [15]. This full space is denoted by $A^N$, where $N$ is the dimension of space, the number of variables (characteristics) used for social system description. In general case $N = \infty$, but for real modeling $N < \infty$. We believe that this space is continuous because the set of attitude is full in the closed region of possible meaning. Mapping the space $A^N$ on the space which corresponds the description of "human" social system is the limited space $H^n$ which appears by the filtration in mean of non-standard analysis [7, 8].

$$H^n = A^N/\aleph \tag{1}$$

$\aleph$ is the filter of transformation of degree $(N - n)$, $n$ is the parameters set of the space of the states $H$. Degree of the filter can be arbitrary, it means that it is possible to widen or narrowing the whole space of the states $A^N$.

**Definition 2**. *In the case $N = n$ the filter $\aleph$ is the exception filter. This filter cuts down characteristics of the homo from the full set of the variables.*

The exact structure of ℵ depends on our understanding of the distinguishing differences between human social structure and organization and other living system social organizations. In such case every social system is determined not only by the variable but by the corresponded filter, too. From the point of view of multilevel hierarchical systems theory [1, 2] the filter is the sway for the space of states.

# 3 Topology of the space of states: hierarchical model of social systems and system evolution

Conditions A-C apply some restriction on possible social system states of space structure. The full topology of space can be obtained by help of the multilevel hierarchical systems theory.

## 3.1 Hierarchical representation

The multilevel hierarchical systems theory (MHST) is developed on the basis of general system theory [17, 18, 19]. The MHST is applied to social systems description and managing [1, 2, 10, 11, 12, 21, 22] for a long time. According to MHST interactions between all systems are divided into two big classes: interactions of control and interactions of collaboration. This two cases of interactions correspond to interactions between systems which are situated at different levels and interactions between systems which are situated at the same level. This way looks like a space of operators for cognitive processes [4]. All systems together form the hierarchical world. This hierarchical world has the graphics image (Fig. 1) [1, 12, 21]. The system development in our space is equal to the movement of a point in the hierarchical space according to general principles of point dynamics in the generalized space [14, 20]. The mathematical images of large scale systems are well known [23]. According to this model our world is the set of interacting strata and interacting systems situated on these strata.

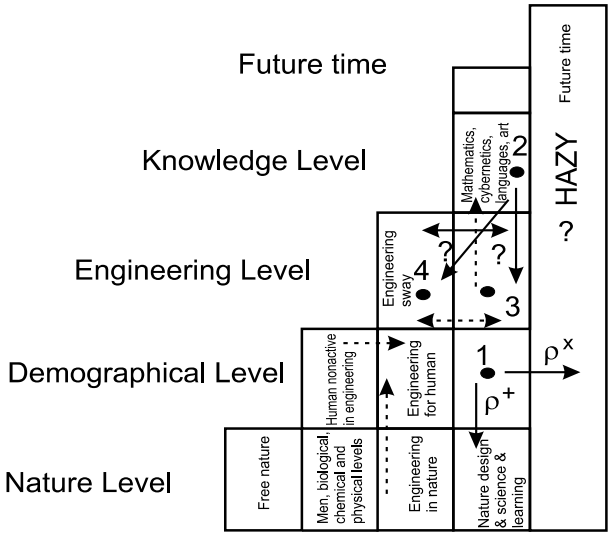


FIG. 1. Short image of the hierarchical world. Quantity $\rho^+$ is the operator of uniting ( transition to new level), $\rho$ is the operator of uniting , solid lines are sway translations, dash lines are signals

The sample of that kind of hierarchical world is the hierarchical State model. The hierarchical representation of world is not isomorphic to natural living pyramid (a.ex. food pyramid in ecology and biology) because they present principally different objects. The food pyramid presents the real objects realized in real space but the hierarchical pyramid presents the object's interactions realized in the space of states. All strata have diverse characteristics in concrete large systems, statutes (symbol image) of which must be connected to their history, with sway strategies in their space. The systems are changing diverse details of their own construction on all strata and these interactions are the base of unions of systems. Till now the process of world changing is realized without actual understanding of its laws and that causes hard mistakes in social system development. The general theory of "human" social system construction and development can be build on the basis of Aed theory.

Aed theory ($A^\lambda$ mathematics, [11]) has now two own main hierarchical symbol images ${}^\times\rho^\lambda$ and ${}^+\rho^\lambda$ which answer to acts of multiplying (learning) and uniting (design). If operator ${}^\times\rho^\lambda$

acting on the system situated on the level (as an example, point 1 on the demographical level) we receive a system, situated on the same level. It is possible to receive a hazy state (state of arising system), too. If operator ${}^{+}\rho^{\lambda}$ acts on point 1, we receive a system situated on the another level. This way the operators ${}^{\times}\rho^{\lambda}$ and ${}^{+}\rho^{\lambda}$ contain the new means of control and connect strata (directions) of $A^{\lambda}$. The full set of aed strata are: $\Lambda$, $\lambda$ is level (time), $\Gamma$, $\gamma$ are statute (law, connection), P, $\rho$ is act (process), $\Omega$, $\omega$ is unit (state) $\Sigma$, $\sigma$ are construction (contents), B, $\beta$ are new time (arising level), A, $\alpha$ is sway (coordinator). All these strata are generalized coordinates, it means different real characteristics of system can be interpreted as a time or some of possible variables may be described as a law. As an example, the condition of dynamical equilibrium for real economical systems is the law variables ( $\Gamma$) for the problem of social system stable development. Aed statute $\mathrm{A}^{\lambda}$ in current level $\lambda$ is described by its symbol image ${}^{\times}\alpha^{\lambda}$ in the following set of locking expression: ${}^{\times}\alpha^{\lambda}$

$$\mathrm{A}^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\mathrm{A}}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\mathrm{A}^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},\quad \Lambda^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\Lambda}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\Lambda^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},$$

$$\Gamma^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\Gamma}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\Gamma^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},\quad \mathrm{P}^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\mathrm{P}}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\mathrm{P}^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},$$

$$\Omega^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\Omega}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\Omega^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},\quad \Sigma^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\Sigma}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\Sigma^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},$$

$$\mathrm{B}^{\lambda}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{\beta}_{\omega}\underset{\rho}{\overset{\gamma}{\mathrm{B}}}{}^{\lambda}_{\sigma}\right\}^{\lambda\overset{\mathrm{B}^{\lambda}}{\underset{\rho}{\rightarrow}}\beta},\quad \mathrm{A}^{\beta}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{?}_{\omega}\underset{\rho}{\overset{\gamma}{\mathrm{A}}}{}^{\beta}_{\sigma}\right\}^{\beta\overset{\mathrm{A}^{\beta}}{\underset{\rho}{\rightarrow}}?} \qquad (2)$$

In this way all aed strata can renovate its original unit $\mathrm{A}^{\lambda}$, they have all its signs and abilities, $\leftrightarrow$ is the correspondence relation. The strata $\Lambda$ , $\Gamma$ , B, P, $\Sigma$, $\Omega$ are strongly connected both by their original unit $\mathrm{A}^{\lambda}$ and by the details of their own constructions (by their new interactions). Thanks to that all aed strata may be renovated when any stratum is changed. The uniting of similar records of all aed details leads to the constructing of highest level unit $A^{\beta}$.

$$\mathrm{A}^{\beta}\underset{\rho}{\overset{\gamma}{\leftrightarrow}}\left\{{}^{?}_{\omega}\underset{\rho}{\overset{\gamma}{\mathrm{A}}}{}^{\beta}_{\sigma}\right\}^{\beta\overset{\mathrm{A}^{\beta}}{\underset{\rho}{\rightarrow}}?} \qquad (3)$$

In this way the original statute aed $A^{\lambda}$ is defined as well as images of its ordinary states ($\Lambda$ , $\Gamma$ , B, P, $\Sigma$, $\Omega$) but the statute of its leading unit $A^{\beta}$ contains in its construction the hazy symbols [25]. It means that new time $\beta$ cannot have exact definition at time $\lambda$

### 3.2 The space mapping

Let study the satisfaction of condition A-C.

**Condition A.** The uncertainties originate from acting the multiplication operator ${}^{\times}\rho$ on the point 1. According to the main properties of hierarchical operators these actions map present point into the state of hierarchical hazy. It means that in the system the hazy occurs. By this mechanism the free will is realized. By this way the local uncertainties of the unit element (the homo) transferring into global uncertainties of the whole system.

**Condition B.** The transition from the point 3 to the point 4 realized by the act of uniting operator ${}^{+}\rho$. The properties of the higher level do not follow from the properties of lower level according to the main mathematical properties of hierarchical space.

**Condition C.** According to the item A, the transition of the system to the hazy region is equal to the system unpredictable behavior. Let us investigate the system situated in the point 4. *At first step* the unit element 4 is changing, this changes the map into the point 3 as a signal of slave system. According to this signal, the system 3 changes its own strategies and structures. By them the control influence (solid lines) from the point 3 organizes the structure of the point 4 according to the task of point 3. Because the task of the coordinator is known only for coordinator, the control influence from the point of view of the system 4 is not completely determined.*Second*

*step.* Because the system 3 changes its own structure and properties the signal transfers to 2 and receives the control influence from the coordinator of the higher level. By them it is possible to direct the influence into 4 or reply only on 3. Because the influence 2-4 is the sway's acting this influence is hidden for the system 3. It means that this influence lies in the hierarchical hazy and acts through the hazy from the point of view of system 3.

### 3.3 Optimal trajectory of the system

Let us study an optimal trajectory of the development of the social system. According to [16] we decide [9] that optimal trajectory will be the stable movement by minimal interaction with other systems.

**Definition 3**. *The condition of minimal interaction corresponds to system movement by the geodesic line in the space of states.*

We have two possible kinds of this movement. The first one is the movement through different levels, the second one is one-level interactions. If $R_g(\{i,j\})$ is the set of states which are equal to the movement by this line, $R(\{i,j\}) \in H^n$ is the set of the states of the system which are equal to the real movement. Here $\{i,j\}$ is the numbered set of hierarchical variables, $i$ are actual for the problem variables, $j$ are other variables. The transition from hierarchical notations to index notations can be made by the different ways. This transition is not important for our problem because sometimes we used short notation, as $R_g(i)$. In this case the problem of local and global optimization of the movement of the system can be formulated:

**Definition 4.** *Local optimality of the development: the development of a social system is locally optimal in the case of equal real and optimal states by the filter of possible deviations*

$$R(i) = R_g(i)/\mathrm{M}, \tag{4}$$

where M is the filter of possible deviations which have the properties

$$\lim_{r\to\infty} (\mathrm{M} - U) = \delta$$

where $r \in H$ is the index of sequence space $H^n$, $U$ is the set of control parameters for a real system, filter for a real social system, $\delta$ is the infinity small value.

**Definition 5.** *Global optimality of the development: social system develops optimally if the set of real states strive to geodesic line between the levels according to the sway law.*

$$\lim_{r\to\infty} (R(i) - R_g(i)) = \delta \tag{5}$$

Equations (4, 5) principally allow to solve the problem of optimization for the development of social systems. We can determine the necessary $U$ for completing the conditions (4, 5) in case we choose $R_g(i)$ and M. Principally we can control $U$ trough sway as a hierarchical structure. It demands from coordinator knowledge about the interaction with other systems of the same level and information exchange.

### 3.4 Problem of collaboration (one-level interaction)

The basis of control of highest stratum is the system of lower stratum. Some different systems are situated at the same time at the same level. The description of this simple system can be developed based on different physical ground: non-equilibrium statistics and thermodynamics, as a system of reaction-diffusion type, information system [5, 6, 14, 15, 16]. As an example, let investigate the systems situated at point 2, 3, 4 on the Figure. The controlling level (point 2, knowledge) influences on both systems, but the results of influence are different because the sway (coordinator) has its own tasks. It is presented by the symbol of hazyness "?" near the control influence (solid lines). This hazyness is realized only from the point of view of the lower level. The bottom levels are effectively influencing on the sway only when deviations of theirs trajectories in hierarchical space from the trajectory

of evolution defined by sway are rather small, the system have local optimum. This influence is realized through information transferring from bottom level to higher level (dashes lines). The value of this deviation $\delta$ for each trajectory is determined by parameters $\Gamma$ and $\gamma$ (law and connection) and is controlled by coordinator. Let us define the full norm in the hierarchical space at present *k-th* level as

$$\left\|\Delta^k\right\| = \sqrt{\sum_{n,i}\left(\left\langle {\Delta_{\{i,j\}|n}}^k\right\rangle - R_g^k(\{i,j\})\right)^2}. \quad (6)$$

Here $\Delta$ is a value of real characteristics, $R_g^k\{i,j\}$ is a value of the characteristics on the geodesic line; in $\Delta^k_{\{i,j\}}$ index $j$ denotes all strata, index $-n$ enumerates all systems which are situated on the current level. We take the trajectory as optimal on the present level when

$$R_g(i)/\mathrm{M^k} < \left\|\Delta^{\mathrm{k}}\right\|. \quad (7)$$

where $\mathrm{M^k}$ is the $k$-layer of full filter M. According to our definition the total trajectory can be optimal in global meaning but not optimal in local meaning due to the big deviation of one of the characteristics.

Let us investigate the interaction between two different system which are situated on the same level. Let us study the case when the hierarchical level contains only two affiliate systems. This intention does not limit the general conclusion. The control is possible only trough the sway, because in the beginning both systems have the equal weight in sway. The first system (point 3) exchanges with the sway its own original unit and details of their own construction. [21, 24]. After that the sway exchanges with the second system (point 4) its own original unit and the details of its own construction. The sway can change the own law because the sway have the law relatively the lower level, too. This exchange is realized by the sway taking into account the information transferred from the lower stratum (dash lines on the picture). The effectiveness of these processes is determined by system competence.

**Definition 6.** *The system is competent if the system of the lower level has mathematical expression of the sway.*

For the competent system this exchange is realized more effective and this system uses law of sway more effective, too. As a result the trajectory of the development of a competent system is nearer to the optimal trajectory and weight of system rises for the sway. Taking into account (6) we can write the partial norm as

$$\left\|\Delta^k_{\{i,j\}|n}\right\| = \sqrt{\sum_{i}\left(\left\langle {\Delta_{\{i,j\}|n}}^k\right\rangle - R_g^k(\{i,j\})\right)^2}. \quad (8)$$

and system is more competent if

$$\left\|\Delta^k_{\{i,j\}|n_1}\right\| < \left\|\Delta^k_{\{i,j\}|n_2}\right\|$$

In any case, interactions between one - level systems are reduced to the interactions between coordinator and the system. Thus the main problem of optimization of the trajectory is the more effective using the law and sway expression.

## 4 The appearance of the uncertainty

For limited space $H^n$ the nature of the appearance of the uncertainty is very interesting. Natural idealization of model of social development is the case of the ideal state of ideal people. It is important that the uncertainty also occurs in this determined system. This situation is equal to the onset of the chaotic regime in the deterministic systems. The beginning of the deterministic chaos is possible in systems with different nature ( physical [3], biological, social [5, 6]). For the simplicity we believe that all social systems are equal in the meaning of nonstandard analysis [6, 8, 9]. It means that in ideal case different social system coincides on the filter do not differ. The usage of the nonstandard analysis and extended universe of variables is caused by the presence of the hierarchical hazy [8] in social system. We cause this hierarchical hazy by our evaluation of systems of higher hierarchical strata.

*It is important to note that the inaccuracy of the uncertainty occurs in the systems with potentially homogeneous equal elements (all people are equal) and potential homogeneous interaction (all are in the boundary of equal rules, economical conditions, etc.).*

### 4.1 Production of uncertainty

While studying the values which are connected with a social system we can assign two big classes: exact-measure and non-exact-measure. Exact-measure values are very seldom (for example the number of representatives in the population at present time) but the other class — non-exact-measure values form the biggest part of parameters which are used in the description of social systems. As a rule all exact-measure values are taking by the calculation the non-exact-measure values. For example, it is possible to determine the exact number of the consumers of this resource now, but future extrapolation of quantity would not be exact.

The hierarchical operators ${}^{+}\rho^{\lambda}$ and ${}^{x}\rho^{\lambda}$ [3, 10] must have nonstandard character because for space $H^n$ hierarchical structure must be a filter. Remember that ${}^{+}\rho^{\lambda}$ create new levels and ${}^{x}\rho^{\lambda}$ is needed in multiplication of current state. According to the definition [7] of a non-standard number, with taking into account the natural algebraic character of direct multiplication and operator nature of ${}^{+}\rho^{\lambda} = t_1 + \tilde{t}_1$ we have:

$$\left({}^{+}\rho^{\lambda_1} +^{+} \rho^{\lambda_2}\right) \mapsto T_1 T_2 = t_1 t_2 + \tilde{t}_1 \tilde{t}_2 \qquad (9)$$

where $\sim$ means halo of standard number and the dimension $H^n$ for the simplification could be taken as two. According to the equation (9) we receive the dynamics of uncertainty for deterministic system by the moving in one level. And power of this uncertainty depends on number of acts of transition operator. If we compare two expressions, (1) and (9) it is easy to see that this mappings are not isomorphic. From the point of view of geometry the operation of filtration builds the mapping of the space $A^N$ on tangent hyperplane $H^n$. Then the equation (1) is a projection of hyper-plane $H^n$ on one of the levels of filter. (It is understanding of the system activity of sway). During this the uncertainty of the state of the system grew as $\tilde{t}_1 \tilde{t}_2 \approx O^2(0)$. And this is natural because the haziness of the state of the coordinator is higher than the haziness of the state of controlled system. Then the action according to the equation (9) is mapping from hyperplane to full space by the filter $F$.

$$A^M = H^n / F \qquad (10)$$

According to equation (1, 10) in general case we obtain

$$A^M \neq A^N \qquad (11)$$

and $A^M$ is the anticipating world. The our system is full competent if we obtain $A^M = A^N$.

## 5 Conclusion

The general equation (2), (3) is the basis for the description of real social system. For detailed description of a given system we need to establish the correspondence between characteristics of system and hierarchical variables $\Gamma, \Omega, \ldots$. Then we obtain the equation of movement in hierarchical space and transfer this equation into the equation for the space of real variables.

The main problem for this task solution is to obtain the full set of variables for given task description. By this way the problem is not the problem of mathematics but the problem of other group of sciences (economics, sociology, demography at all...). Thus as a conclusion we obtain some general points:

- The problem of social system description is connected with the exact representation of the set of system variables. The full state space can be build on the basis of non-standard approximation of unit elements of MHST.
- The social system evolution can be represented as a movement in the hierarchical space of state.

- The appearance of the social system development uncertainty can be connected with the non-standard value of social characteristics set.

- According to own sway task the system can develop with local but not with global optimum. This meet the case when local geodesic line in the space of states (geodesic line on the present hierarchical stratum) do not coincide with the global geodesic line (geodesic line pass trough some strata).

## Acknowledgement

Author is grateful to the Dr. V. Barkaline for the valuable comments and S. Miklashevich for the help in preparing this work.

### References

[1] S. Novikava, K. Miatliuk, S. Gancharova and V. Kaliada *Aed Construction and Technology in Design*, in Proceedings of the 7th IFAC Symposium on Large Scale Systems: Theory and Applications LSS'95, London, UK, 1995, pp.379-381.(1995).

[2] I. Miklachevich, *Modeling and optimal control in ethnosocial systems*, in $18^{Th}$ Int. System Dynamics Conference "Sustainability in the Third Millenium", Bergen, Norway, August 6-10, p.145. (2000).

[3] Kuchtenko A. I. Analytical theory of logical - dynamical systems. *Preprint 94-20, Institute of cybernetics Ukrainian Academy of Science.* (Kiev, 1994).

[4] D. Gernet BioSystems. **54**, 165-180 (2000).

[5] H.G. Schuster. *Deterministic Chaos.* (Veincheim, VCN, 1995).

[6] D.S. Dendrinos and M. Sonis. *Chaos and socio-spatial dynamics.* (Springer, 1990).

[7] S. Albeverio , J.E. Fenstad, R. Høegh-Krohn, T. Lindstrøm. *Nonstandard Methods in Stochastic Analysis and Mathematical Physics.* (Orlando/San Diego, Academic Press, 1986).

[8] A.G. Kisraev , S.S. Kutateladze. *Nonstandard Methods of Analysis.* (Academic Press, Novosibirsk, 1990).

[9] I. Miklashevich. Dynamics of haze in hierarchical systems. *Int. Conf. Of System Thinking in Management.* (Deakin University, Geelong, Australia $8^{th}$ -$10^{th}$ Nov. 2000).

[10] S. Novikava, G. Ananich, and R. Miatliuk, *The Structure and the Dynamics of Information in Design Systems*. In: Proceedings of 7th Conference on Engineering Design, ICED'90, Dubrovnik, **2**, 946-953 (1990).

[11] S. Novikava, K. Miatliuk, et all *Aed Technology for Ecological, Social and Engineering Systems Coordination*. In: Proceedings of 8th International Symposium on Modular Information Computer Systems and Networks, ICSNET'91, Dubna, Russia, pp.145-152 (1991).

[12] S. Novikava, et al. *Hierarchical Multilevel Systems in Aed Realization*. in: Proceedings of 9th International Conference on Mathematical and Computer Modelling, ICMCM'93, p.71, Berkeley, USA. (1993).

[13] A. Khrennivov. Foundations of Physics. **29**, no. 7, 1065-1098 (1999).

[14] W. Weidlich. Phys. Rep. **204**, no.1, 1-166 (1991).

[15] W. Weidlich, G. Haag. *Concepts and Models of a Quantitative Sociology. Springer Series in Synergetics.* **.14**. (Heidelberg, Berlin. Springer-Verlag. New York. 1983).

[16] D. Helbing, T. Vicsek. New Journal of Physics. **1**, no. 13, 1-13 (1999).

[17] M.D. Mesarovic, D. Masko, Y. Takahara Y. *Theory of Hierarchical Multilevel Systems.* (Academic Press, New York and London, 1970).

[18] M.D. Mesarovic, Y. Takahara. *General Systems Theory: Mathematical Foundation.* (Academic Press, New York and London, 1975).

[19] L. von Bertalanffy, W. Beier, R. Lane *Biophysik des Fließgleichgewichts.* (Akademie Verlag, Berlin, 1977).

[20] H.J. Bremermann, *On the Dynamics and the Trajectories of Evolution Processes.* (Wien-Heidelberg-N.Y, 1972).

[21] P. Buka, A. Konash, A. Siargeichyk A, et al. The Constructing of New Electronic Device with Aed Processor. *Preprints of Eleventh International Conference on Mathematical and*

*Computer Modelling and Scientific Computing.* (Washington, DC, USA. p.47, 1997).

[22] E.M. Blanter, M.G. Scnirman M.G. Physical Review E. **55**, no.6, 6397 - 6403 (1997).

[23] H. Chestnut *Improving International Stability and Maintaining Peace* . In: Preprints of the IFAC Conference on Supplementary Ways for Improving International Stability, SWIIS'95, Vienna, Austria, p.11-14, (1995).

[24] S. Novikava, S. Gancharova, A. Zhybul, et al. *The Statute of Hierarchical Mathematics and Its Cybernatical Maintenance*. In: Preprints of Eleventh International Conference on Mathematical and Computer Modelling and Scientific Computing. (Washington, DC, USA. p.149. (1997).

[25] W. Nowik, V. Barkalin et all. *Demographical units in hierarchical coordinates: construction, activity and goals.* In: Proceedings of the IFAC Conference on Supplementary Ways for Improving International Stability, SWIIS'98, May 14-16, Sinaia, Romania, p.95-100, (1998).